Interval Analysis of Linear Analog Circuits

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Abstract

Reliable methods for the analysis of tolerance affected analog circuits are of great importance in nowadays microelectronics. It is impossible to produce circuits with exactly those parameter specifications proposed in the design process. Such component tolerances will always lead to small variations of a circuit's properties, which may result in unexpected effects. If lower and upper bounds to parameter variations can be read off the manufacturing process, interval arithmetic naturally enter the circuit analysis area.

The following focusses on the analysis of linear analog circuits, typically consisting of current and voltage sources as well as resistors, capacitances, and inductivities. These are still widely used in analog circuit design as equivalent circuit diagrams for representing more complex systems in certain applications like frequency domain analysis **[1]**.

Interval methods have been applied to analog circuits before. But yet this was restricted to circuit equations, with no interdependencies between the matrix elements **[2]**. The latter is not true for all formulations of the analog circuit equations. Hence, for an efficient application of interval methods, it is crucial to regard possible dependencies in circuit equations. Part and parcel of of this strategy is the handling of fill-in patterns for those parameters related to uncertain components. These patterns are used in linear circuit analysis for efficient equation setup.

For purely resisitive circuits the fill-in pattern of an uncertain parameter p corresponds to a rank-one update of a real-valued matrix. This perfectly fits to the application of a suitable variant of the *Sherman-Morrison Formula*

$$\left(\mathbf{A} + p\,\mathbf{u}\cdot\mathbf{v}^{\mathsf{T}}\right)^{-1}\mathbf{b} = \mathbf{A}^{-1} - \frac{1}{1/p + \mathbf{v}^{\mathsf{T}}\mathbf{A}^{-1}\mathbf{u}}\mathbf{A}^{-1}\mathbf{u}\mathbf{v}^{\mathsf{T}}\mathbf{A}^{-1}\mathbf{b}\,,$$

for real-valued vectors $\mathbf{u}$, $\mathbf{v}$ defining the fill-in pattern, and right-hand side $\mathbf{b}$. This expression can be evaluated sharply for interval-valued p. For n independent parameters sufficiently narrow outer approximations can be obtained by successive application of the formula. Since also auxiliary values like $\mathbf{A}^{-1}\mathbf{u}$ have to be updated during the procedure, this leads

to an algorithm, whose number of steps is of complexity order $\mathcal{O}(n^2)$ [**3**]. Since a large number of affected components may be treated, analysis of real-world devices is rendered possible.

The approach can also be extended to complex-valued systems from frequency domain analysis of more general linear circuits. Complex values result here from a Laplace transform of frequency-dependent components like capacitances and inductivities. In order to apply interval techniques, a real representation of the linear system of equations can be used for separate treatment of real and imaginary part of the variables. In this representation each parameter corresponds to the superposition of two fill-in patterns. Crude bounds – obtained by treating both patterns independently – can be improved by consideration of the correlations to tighter enclosures of the solution.

The techniques described above have been implemented as an extension to the toolbox *Analog Insydes* [**4**], an add-on package to the computer algebra system *Mathematica* [**5**] for modeling, analysis, and design of analog circuits.

References:

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