Fraunhofer Institut Techno- und Wirtschaftsmathematik

M. T. Melo, S. Nickel, F. Saldanha da Gama

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Berichte des Fraunhofer ITWM, Nr. 58 (2003)

ISSN 1434-9973

Bericht 58 (2003)

Die Veröffentlichungen in der Berichtsreihe des Fraunhofer ITWM können bezogen werden über:

Fraunhofer-Institut für Techno- und
Wirtschaftsmathematik ITWM
Gottlieb-Daimler-Straße, Geb. 49

67663 Kaiserslautern
Germany

Telefon: +49 (0) 6 31/2 05-32 42
Telefax: +49 (0) 6 31/2 05-41 39
E-Mail: info@itwm.fraunhofer.de
Internet: www.itwm.fraunhofer.de

Vorwort

Das Tätigkeitsfeld des Fraunhofer Instituts für Techno- und Wirtschaftsmathematik ITWM umfasst anwendungsnahe Grundlagenforschung, angewandte Forschung sowie Beratung und kundenspezifische Lösungen auf allen Gebieten, die für Techno- und Wirtschaftsmathematik bedeutsam sind.

In der Reihe »Berichte des Fraunhofer ITWM« soll die Arbeit des Instituts kontinuierlich einer interessierten Öffentlichkeit in Industrie, Wirtschaft und Wissenschaft vorgestellt werden. Durch die enge Verzahnung mit dem Fachbereich Mathematik der Universität Kaiserslautern sowie durch zahlreiche Kooperationen mit internationalen Institutionen und Hochschulen in den Bereichen Ausbildung und Forschung ist ein großes Potenzial für Forschungsberichte vorhanden. In die Berichtreihe sollen sowohl hervorragende Diplom- und Projektarbeiten und Dissertationen als auch Forschungsberichte der Institutsmitarbeiter und Institutsgäste zu aktuellen Fragen der Techno- und Wirtschaftsmathematik aufgenommen werden.

Darüberhinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation darüber, wie aktuelle Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte transferiert werden, und wie umgekehrt Probleme der Praxis neue interessante mathematische Fragestellungen generieren.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

Large-scale models for dynamic multi-commodity capacitated facility location

M.T. Melo[a], S. Nickel[a,b,*], F. Saldanha da Gama[c]

[a] *Department of Optimization, Fraunhofer Institute for Industrial Mathematics (ITWM), D 67663 Kaiserslautern, Germany*
[b] *Chair of Operations Research and Logistics, Saarland University, D 66041 Saarbrücken, Germany*
[c] *Department of Statistics and Operations Research, University of Lisbon, P 1749-016 Lisbon, Portugal*

Abstract

In this paper we focus on the strategic design of supply chain networks. We propose a mathematical modeling framework that captures many practical aspects of network design problems simultaneously but which have not received adequate attention in the literature. The aspects considered include: dynamic planning horizon, generic supply chain network structure, external supply of materials, inventory opportunities for goods, distribution of commodities, facility configuration, availability of capital for investments, and storage limitations. Moreover, network configuration decisions concerning the gradual relocation of facilities over the planning horizon are considered. To cope with fluctuating demands, capacity expansion and reduction scenarios are also analyzed as well as modular capacity shifts. The relation of the proposed modeling framework with existing models is discussed. For problems of reasonable size we report on our computational experience with standard mathematical programming software. In particular, useful insights on the impact of various factors on network design decisions are provided.

Keywords: supply chain management, strategic planning, dynamic location, modeling

1 Introduction

A supply chain is a network of facilities (e.g. manufacturing plants, distribution centers, warehouses, etc.) that performs a set of operations ranging from the acquisition of raw materials, the transformation of these materials into intermediate and finished products, to

*Corresponding author. *E-mail address*: nickel@itwm.fhg.de

the distribution of the finished goods to the customers (see e.g. Bramel and Simchi-Levi [6]). Figure 1 shows a supply chain network with suppliers, plants, distribution centers (DCs) and customers. The arrows indicate the transportation channels that are available to ship products. In addition to inbound and outbound transportation, products may flow between facilities of the same type and from customer locations to other facilities (e.g. plants, DCs). The latter material flows describe product recovery activities for the purposes of recycling, remanufacturing and re-use. Such activities build the field of reverse logistics which has attracted considerable attention over the last decade (see e.g. Fleischmann [12]).

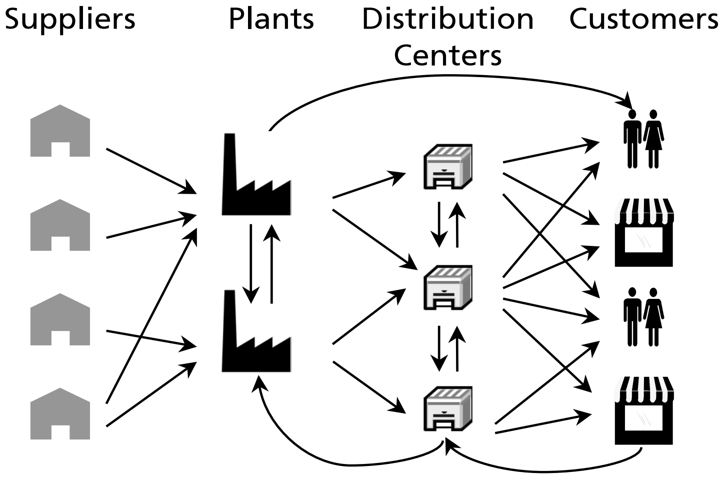

Figure 1: Example of a supply chain network.

The optimization of the complete supply chain is accomplished through efficient planning decisions. In the strategic planning level, typical decisions concern the location of manufacturing and/or warehousing facilities. Although facility location and configuration of production–distribution networks have been studied for many years (see Bender *et al.* [5] for a review of models, company-specific case studies, and decision support systems), a number of important real world issues has not received adequate attention. These include, among others, the external supply of commodities, inventory opportunities for goods, storage limitations, availability of capital for investments, and relocation, expansion or reduction of capacities. In addition, the supply chain network typically consists of arborescent structures usually limited to at most two echelons of facilities (e.g. plants and DCs), a system of distribution channels between these echelons and the demand points, and a relatively simple cost structure. Even though some of these issues have been treated individually in the literature, our extensive experience with a variety of industrial projects in the supply chain planning area indicates that companies wish them all to be explicitly incorporated in the design of their supply chain networks (see e.g. Bender *et al.* [5], Kalcsics *et al.* [25] and the website of Fraunhofer ITWM [19] for an overview of recent projects). Clearly, the structure of a network is strongly affected by the simultaneous consideration of these and other practical needs. One observes a lack of reasonably simple, yet comprehensive, models which illustrate the effect of such factors on network configuration decisions. This paper attempts to fill this gap. Our main contribution is to provide a mathematical modeling framework for assisting decision-makers in the design of their supply chain networks.

The establishment of new facilities as well as the expansion, reduction and relocation of existing facilities are typically long–term projects involving time-consuming activities (e.g., facility constructing) and substantial investment capital (e.g., installation of an adequate infrastructure, equipment supply and employee training). As a result, companies are faced with the task of planning their supply chains in such a way that plans are carried out smoothly without disrupting supply chain activities. For example, in many manufacturing environments the setup or relocation of a production facility is a lengthy project which requires time-phased planning. A smooth transition to a new network configuration allows better coordination of all operational aspects involved in this process, and better management of the required investment capital. Hence, to abate the financial burden put on the company by such a comprehensive project, capital expenditures as well as network design decisions should be planned over several time periods. Another practical situation to which a relocation scenario applies concerns the merger of companies. In this case, formerly separated supply chains are consolidated and a joint logistics structure needs to be planned. This entails shutting down some of the existing facilities and concentrating capacities in new locations.

In this paper we address the dynamic location and relocation of facilities, and propose a mathematical modeling framework to tackle such problems. As will be shown, our approach not only deals with the gradual relocation of existing facilities but also comprises capacity expansion and capacity reduction scenarios. Furthermore, our model includes many dynamic facility location problems, which have appeared in the literature, as special cases. Our primary concern is to provide a comprehensive modeling framework which explicitly captures many practical aspects that play an important role in shaping the optimal structure of supply chain networks. In particular, our models focus on the following strategic issues:

- Where and when should a partial or total relocation of existing facilities take place?
- Using the available investment capital and considering the capacity limitations, how gradually should the capacity of existing facilities be reduced and when should the transition to newly established facilities be made without disrupting the supply chain activities?
- How should the supply chain be operated in each planning period with respect to the provision of materials, storage and shipment of products so that demand requirements are satisfied at least cost?

Before concluding the introduction, a few remarks are in order concerning how our modeling framework fits into the existing literature. The literature devoted to dynamic facility location in a multi-commodity, multi-level (or multi-echelon) supply chain network is relatively scarce. However, a great deal of research has been dedicated to some of these aspects individually and to specific combinations of them.

Since the pioneering work by Ballou [2], many papers addressing the dynamic facility location problem have appeared. The uncapacitated case was studied, amongst others, by Chardaire *et al.* [9], Galvão and Santibañez-Gonzalez [15], Kelly and Marucheck [27], Khumawala and Whybark [28], Roodman and Schwarz [34, 35], and Van Roy and Erlenkotter [40]. Furthermore, Canel and Khumawala [7] considered the dynamic location of international facilities, while Saldanha da Gama [36] incorporated budget constraints in his models. Wang *et al.* [42] also introduced a budget constraint into a static location problem.

Concerning the capacitated version of the problem, the possibility of expanding / reducing the operating capacity is often considered. One of the earliest attempts to incorporate capacity constraints is due to Sweeney and Tatham [38]. Erlenkotter [11], Fong and Srinivasan [13, 14], Jacobsen [23], and Lee and Luss [29] analyzed the problem of deciding in which periods the capacity of a given set of facilities should be extended to meet increasing demand patterns. Antunes and Peeters [1] and Shulman [37] addressed the case of modular capacities. Melachrinoudis *et al.* [31] studied a multi-objective version of the capacitated multi-period location problem.

The problem of deciding where to locate a set of intermediate facilities (such as DCs) was examined by Elson [10]. Geoffrion and Graves [16] and, more recently, Hormozi and Khumawala [20] studied the same problem. These models were later extended leading to the problem of locating more than one echelon of facilities. Barros [3], Barros and Labbé [4], Kaufman *et al.* [26], Pirkul and Jayaraman [33], and Tcha and Lee [39] have dealt with the static version of this problem.

The combination of dynamic and multi-commodity aspects in the problem of deciding the best location for a set of intermediate capacitated facilities was addressed by Canel *et al.* [8]. Melachrinoudis and Min [30] and Min and Melachrinoudis [32] focused on the relocation of one single intermediate facility in a multi-objective context. Finally, Hinojosa *et al.* [18] studied a problem combining dynamic aspects with multi-level facility location in a multi-commodity distribution network. This problem was later extended by Velten [41] to integrate inventory considerations. Inventories were also explicitly considered by the dynamic model proposed by Gue [17] for combat logistics.

The models to be presented in Sections 3 and 4 go beyond the literature described above by considering a broader context for facility location and relocation. This means that many issues that are likely to occur in practical settings (see [19]) are explicitly taken into account. These include a dynamic planning horizon, a generic network structure, the planning of supply chain activities (i.e. production, inventory, distribution) in addition to facility location/relocation, and capacity and budget constraints. As mentioned before, we will focus on modeling rather than on algorithmic aspects.

The remainder of the paper is organized as follows. Section 2 describes the general settings and assumptions used in our modeling approach. In Section 3, a mixed integer linear programming formulation is presented for the dynamic relocation of facilities. Section 4 is dedicated to extensions of this base model. We address capacity expansion and reduction scenarios as well as the case of discrete capacity sizes. To ease the exposition, it is shown in Appendix A that our modeling framework also includes the simple dynamic facility location problem (of the type e.g. analyzed by Van Roy and Erlenkotter [40]) as a special case. Section 5 emphasizes the way in which the new framework generalizes models widely known from the literature, and also gives a brief review of the techniques that have been used to deal with them. Section 6 reports on the computational experience with the proposed models using commercially available mathematical programming software. In particular, useful insights on the impact of various factors on network design decisions are provided. Finally, Section 7 presents some concluding remarks and suggests directions for future research.

2 Problem settings and description

To provide a setting for our modeling framework, we consider a general supply chain network where different products are delivered to satisfy the requirements of several demand points (henceforth also called facilities). The network may accommodate different types of facilities (e.g. plants, DCs, warehouses), which are already operating at the beginning of the planning project. No restrictions are imposed on the number of different facility types and on the transportation links used by the company for shipping its products. In other words, commodities can be transported between any type of facility as illustrated in Figure 1. Moreover, supply chain operations in such a network are mainly dedicated to distribution activities with the purpose of satisfying known demand requirements. Therefore, due to the general structure of the supply chain model, to ensure that products will flow through the network, a source for their distribution needs to be set. As a result, either products are produced in certain pre-specified facilities or purchased from outside suppliers. In the former case, and due to the strategic nature of the problem, a complete description of the underlying production system in each facility is not considered. In other words, only production decisions regarding end products are taken into account. Hence, requirements for components, subassemblies and raw materials as commonly described by the bill-of-materials of end products are not included in our models. In addition to production and procurement, also inventory activities are considered. Observe that fluctuations in production, procurement and transportation costs along with capacity constraints may justify holding products in stock for later distribution.

We assume that a company is considering relocating part or all of the capacity of some of its existing facilities during a certain time horizon. Prior to the planning project, the company has selected a set of candidate sites where new facilities can be established. Furthermore, the existing capacities as well as the capacities of the prospective sites are known in advance. In many applications, management does not wish to open a facility unless its projected output is some minimum level. Therefore, when capacity transfers occur they must ensure that both existing and new facilities operate at least at a meaningful level.

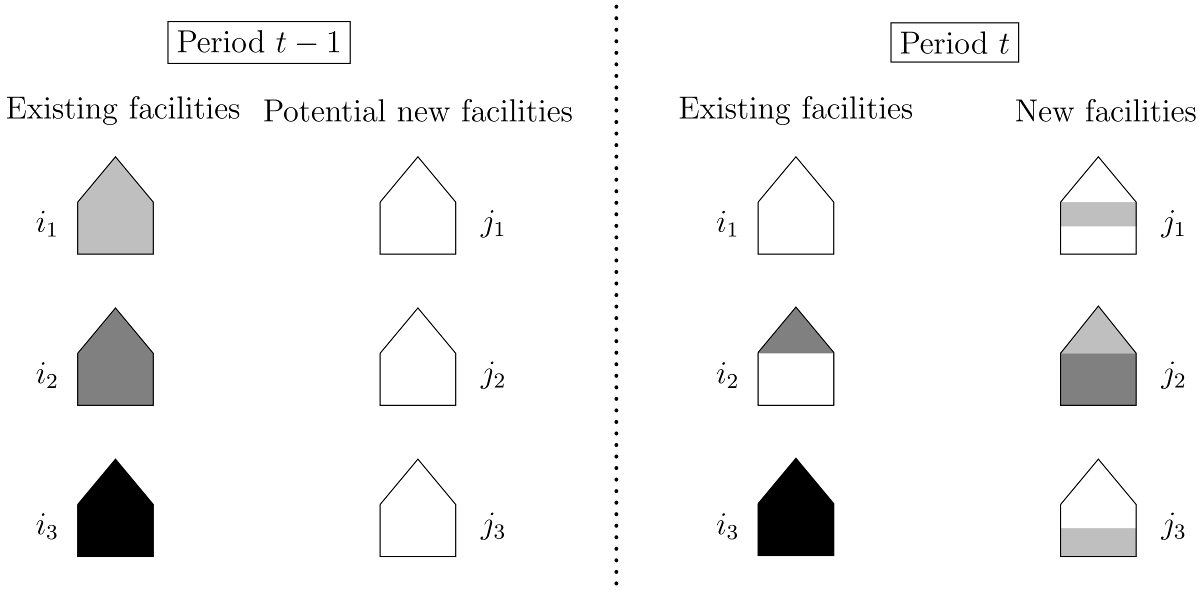

Figure 2: The effect of capacity relocation.

2.1 Capacity relocation

To illustrate the conditions under which capacity may be transferred from existing locations to new sites, consider the situation depicted in Figure 2. At the end of a given period, say $t-1$, the company operates facilities i_1, i_2, and i_3. Potential sites for setting up new facilities are denoted by j_1, j_2, and j_3. It is assumed that if capacity is to be shifted then this will occur at the beginning of period t, and will have a relatively short duration compared to the period length. On the right-hand side of Figure 2 a possible scenario for capacity shifting is displayed. At the beginning of period t new facilities will be operating in all three potential sites. The capacity of the existing facility i_1 is distributed among the sites j_1, j_2, and j_3. As a result, facility i_1 will neither operate in period t nor in any subsequent period. Thus, we assume that the facility is closed at the end of period $t-1$, and its total capacity is moved to the new sites at the beginning of period t. Part of the capacity of the existing facility i_2 is transferred to the new site j_2. Facility i_2 remains in operation but with reduced capacity. Note that the new facility j_2 has attained its maximum capacity, while facilities j_1 and j_3 can still have their capacity extended in later periods. Finally, the existing facility i_3 retains all its capacity.

As shown in Figure 2, an existing facility may be partly or completely relocated to one or more new sites. Capacity transferred to a new facility cannot be removed in later periods, that is, capacity reductions are only allowed at existing facilities. Under increasing demand patterns during the planning horizon, it may be necessary to extend the total capacity available in the system at the beginning of the relocation project. This is accomplished by permitting capacity expansions in both existing and new facilities. Section 4.1 will be dedicated to this scenario. The opposite case will be treated in Section 4.2.

2.2 Cost factors

The majority of facility location models seeks to minimize the total distribution costs, both inbound (e.g., from plants to DCs) and outbound costs (e.g., from DCs to demand points) plus fixed facility operating and setup costs. In our case, costs are divided into two categories: the so-called *business costs* which result from operating the supply chain, and *investment costs* for facility relocation, which are constrained by the available budget.

The first category comprises time dependent costs for the purchase of products from external suppliers, production costs, transportation costs, inventory holding costs and fixed facility operating costs, e.g. fixed overhead and maintenance costs. The objective is to find the relocation plan that minimizes the sum of the business costs under given side constraints.

As mentioned above, relocation decisions are constrained by budget limitations. We assume that in each time period a given budget is available for investing not only in capacity transfers, but also in the setup of new facilities and shutdown of existing facilities. Relocation costs incurred by capacity shifts are assumed to depend on the amount moved from an existing facility to a new site, and account, for example, for workforce and equipment transfers. Since the setup of a new facility is usually a time-consuming process, we assume that it takes place in the period immediately preceding the start-up of operations. Hence, if a new facility starts operating in some period t, fixed costs are charged with respect to its setup in period $t-1$. These costs account for the installation of a new infrastructure and include, for instance, property acquisition, facility construction and traffic access. On the other hand, when an existing facility ceases operating at the end of some period t, we assume that shutdown costs are incurred in the following period. These are related to disposal activities but may also cover other aspects such as employee re-training. Observe that in certain situations shutdown costs may be negative, that is, may correspond to revenues obtained, for example, due to the termination of leasing contracts or the selling of property.

In view of the assumptions made on the time points for paying fixed facility costs, it follows that a new facility can never start operating in the first period since that would force the company to invest in its setup before the beginning of the planning horizon. As a result, capacity transfers are only allowed in periods after the first. Similarly, an existing facility cannot be closed at the end of the last period since the corresponding fixed shutdown costs would be incurred in a period beyond the planning horizon. Finally, any capital available in a period but not invested then, is subject to an interest rate and the returned value can be used in subsequent periods.

3 Problem formulation

Before presenting a mixed integer linear programming model (MIP) for the dynamic relocation problem described in Section 2, we first introduce the notation that will be used throughout the paper. We assume that prior to the relocation project, all relevant data (costs, capacities and other factors) were collected using e.g. appropriate forecasting methods and company specific business analyzes.

3.1 Notation and definition of decision variables

Index sets

$\mathcal{L}$: set of facilities

$\mathcal{S}$: set of *selectable* facilities, $\mathcal{S} \subset \mathcal{L}$

$\mathcal{S}^c$: set of *selectable existing* facilities, $\mathcal{S}^c \subset S$

$\mathcal{S}^o$: set of potential sites for establishing *new* facilities, $\mathcal{S}^o \subset \mathcal{S}$

$\mathcal{P}$: set of product types

$\mathcal{T}$: set of periods

Set $\mathcal{L}$ contains all types of facilities. These are categorized in so-called *selectable* and *non-selectable* facilities. Selectable facilities form the set $\mathcal{S}$, a subset of $\mathcal{L}$, and include existing facilities ($\mathcal{S}^c$) as well as potential sites for establishing new facilities ($\mathcal{S}^o$). At the beginning of the planning horizon, all the facilities in the set $\mathcal{S}^c$ are operating. Afterwards, capacity can be shifted from these facilities to new facilities located at the sites in $\mathcal{S}^o$. Note that $\mathcal{S}^c \cap \mathcal{S}^o = \emptyset$ and $\mathcal{S}^c \cup \mathcal{S}^o = \mathcal{S}$.

The second category of facilities, the so-called non-selectable group, forms the set $\mathcal{L} \setminus \mathcal{S}$ and includes all facilities that exist at the beginning of the planning project and which will remain in operation. Examples of such facilities include plants and warehouses which should continue supporting supply chain activities, that is, are not subject to relocation decisions. Non-selectable facilities may also have demand requirements, that is, they may correspond to customers.

The planning horizon is partitioned into a set of consecutive and integer time periods which may not necessarily have equal size. In total there are n planning periods.

Costs

$PC^t_{\ell,p}$	:	variable cost of purchasing or producing one unit of product $p \in \mathcal{P}$ by facility $\ell \in \mathcal{L}$ in period $t \in \mathcal{T}$
$TC^t_{\ell,\ell',p}$	:	variable cost of shipping one unit of product $p \in \mathcal{P}$ from facility $\ell \in \mathcal{L}$ to facility $\ell' \in \mathcal{L}$ ($\ell \neq \ell'$) in period $t \in \mathcal{T}$
$IC^t_{\ell,p}$	:	variable inventory carrying cost per unit on hand of product $p \in \mathcal{P}$ in facility $\ell \in \mathcal{L}$ at the end of period $t \in \mathcal{T}$
$MC^t_{i,j}$	:	unit variable cost of moving capacity from the existing facility $i \in \mathcal{S}^c$ to a new facility established at site $j \in \mathcal{S}^o$ at the beginning of period $t \in \mathcal{T} \setminus \{1\}$
OC^t_ℓ	:	fixed cost of operating facility $\ell \in \mathcal{L}$ in period $t \in \mathcal{T}$
SC^t_i	:	fixed cost charged in period $t \in \mathcal{T} \setminus \{1\}$ for having shut down the existing facility $i \in \mathcal{S}^c$ at the end of period $t-1$
FC^t_j	:	fixed setup cost charged in period $t \in \mathcal{T} \setminus \{n\}$ when a new facility established at site $j \in \mathcal{S}^o$ starts its operation at the beginning of period $t+1$

Parameters

$\overline{K}^t_\ell$	:	maximum allowed capacity at facility $\ell \in \mathcal{L}$ in period $t \in \mathcal{T}$
$\underline{K}^t_\ell$	:	minimum required throughput at the selectable facility $\ell \in \mathcal{S}$ in period $t \in \mathcal{T}$
$\mu_{\ell,p}$	:	unit capacity consumption factor of product $p \in \mathcal{P}$ at facility $\ell \in \mathcal{L}$
$H_{\ell,p}$	:	stock of product $p \in \mathcal{P}$ at facility $\ell \in \mathcal{L}$ at the beginning of the planning horizon (observe that $H_{\ell,p} = 0$ for every $\ell \in \mathcal{S}^o$)
$D^t_{\ell,p}$	:	external demand of product $p \in \mathcal{P}$ at facility $\ell \in \mathcal{L}$ in period $t \in \mathcal{T}$
β^t	:	interest rate in period $t \in \mathcal{T}$

α^t : unit return factor on capital not invested in period $t \in \mathcal{T} \setminus \{n\}$, that is, $\alpha^t = 1 + \beta^t/100$

B^t : available budget in period $t \in \mathcal{T}$

Since each existing facility belonging to the selectable set $\mathcal{S}^c$ may have its capacity transferred to one or more new facilities, it is assumed that its maximum capacity is non-increasing during the planning horizon, that is,

$$\overline{K}_i^t \geq \overline{K}_i^{t+1}, \qquad i \in \mathcal{S}^c,\, t \in \mathcal{T} \setminus \{n\}\,. \tag{1}$$

Without loss of generality, it is assumed that $\overline{K}_i^1$ denotes the actual size of an existing selectable facility $i \in \mathcal{S}^c$ at the beginning of the planning horizon. Condition (1) permits to impose capacity transfers in specific periods or even complete shutdowns. One simply needs to decrease the maximum allowed size of the existing facility for which a reduction is desired or even set the capacity equal to zero in the case of a shutdown.

Similarly, potential new facilities must have non-decreasing capacities during the planning horizon, that is,

$$\overline{K}_j^t \leq \overline{K}_j^{t+1}, \qquad j \in \mathcal{S}^o,\, t \in \mathcal{T} \setminus \{n\}\,. \tag{2}$$

Clearly, at the beginning of the planning project we have that $\overline{K}_j^1 = 0$ for every new site $j \in \mathcal{S}^o$. In addition, by allowing time-dependent capacity levels, seasonal demand fluctuations can easily be modeled. For example, in the beverage sector, demand usually grows during the summer thus leading to an increase in the required number of working shifts or in the floor space hired for storing products.

Finally, we note that since non-selectable facilities will remain in operation over the time horizon, independent of their product flow, a minimum throughput is not specified for them. However, this feature can easily be added to our model.

Decision variables

$b_{\ell,p}^t$ = amount of product $p \in \mathcal{P}$ produced or purchased from an outside supplier by facility $\ell \in \mathcal{L}$ in period $t \in \mathcal{T}$

$x_{\ell,\ell',p}^t$ = amount of product $p \in \mathcal{P}$ shipped from facility $\ell \in \mathcal{L}$ to facility $\ell' \in \mathcal{L}$ $(\ell \neq \ell')$ in period $t \in \mathcal{T}$

$y_{\ell,p}^t$ = amount of product $p \in \mathcal{P}$ held in stock in facility $\ell \in \mathcal{L}$ at the end of period $t \in \mathcal{T} \cup \{0\}$ (observe that $y_{\ell,p}^0 = H_{\ell,p}$)

$z^t_{i,j}$ = amount of capacity shifted from the existing facility $i \in \mathcal{S}^c$ to a newly established facility at site $j \in \mathcal{S}^o$, at the beginning of period $t \in \mathcal{T}$

ξ^t = capital not invested in period $t \in \mathcal{T}$

$$\delta^t_\ell = \begin{cases} 1 & \text{if the selectable facility } \ell \in \mathcal{S} \text{ is operated during period } t \in \mathcal{T} \\ 0 & \text{otherwise} \end{cases}$$

Since capacity transfers are not permitted at the beginning of the planning horizon, it follows that $z^1_{i,j} = 0$ for every $i \in \mathcal{S}^c$ and $j \in \mathcal{S}^o$ (recall Section 2.2). Actually, in the first period all existing facilities are operating, that is, $\delta^1_i = 1$ with $i \in \mathcal{S}^c$, and new facilities cannot be established, that is, $\delta^1_j = 0$ with $j \in \mathcal{S}^o$.

3.2 Formulation

The MIP formulation of our model is as follows.

$$(P) \quad \text{MIN} \sum_{t\in\mathcal{T}}\sum_{\ell\in\mathcal{L}}\sum_{p\in\mathcal{P}} PC^t_{\ell,p}\, b^t_{\ell,p} + \sum_{t\in\mathcal{T}}\sum_{\ell\in\mathcal{L}}\sum_{\ell'\in\mathcal{L}\setminus\{\ell\}}\sum_{p\in\mathcal{P}} TC^t_{\ell,\ell',p}\, x^t_{\ell,\ell',p}$$
$$+ \sum_{t\in\mathcal{T}}\sum_{\ell\in\mathcal{L}}\sum_{p\in\mathcal{P}} IC^t_{\ell,p}\, y^t_{\ell,p} + \sum_{t\in\mathcal{T}}\sum_{\ell\in\mathcal{S}} OC^t_\ell\, \delta^t_\ell + \sum_{t\in\mathcal{T}}\sum_{\ell\in\mathcal{L}\setminus\mathcal{S}} OC^t_\ell \tag{3}$$

s.t.

$$b^t_{\ell,p} + \sum_{\ell'\in\mathcal{L}\setminus\{\ell\}} x^t_{\ell',\ell,p} + y^{t-1}_{\ell,p} = D^t_{\ell,p} + \sum_{\ell'\in\mathcal{L}\setminus\{\ell\}} x^t_{\ell,\ell',p} + y^t_{\ell,p},\ \ell \in \mathcal{L},\ p \in \mathcal{P}, t \in \mathcal{T} \tag{4}$$

$$\overline{K}^1_i - \sum_{\tau=1}^{t}\sum_{j\in\mathcal{S}^o} z^\tau_{i,j} \le \overline{K}^t_i\, \delta^t_i, \qquad i \in \mathcal{S}^c,\ t \in \mathcal{T} \tag{5}$$

$$\sum_{\tau=1}^{t}\sum_{i\in\mathcal{S}^c} z^\tau_{i,j} \le \overline{K}^t_j\, \delta^t_j, \qquad j \in \mathcal{S}^o,\ t \in \mathcal{T} \tag{6}$$

$$\sum_{\tau=1}^{t}\sum_{j\in\mathcal{S}^o} z^\tau_{i,j} + \delta^t_i\, \epsilon \le \overline{K}^1_i, \qquad i \in \mathcal{S}^c,\ t \in \mathcal{T} \tag{7}$$

$$\sum_{p\in\mathcal{P}} \mu_{i,p}\left(b^t_{i,p} + \sum_{\ell\in\mathcal{L}\setminus\{i\}} x^t_{\ell,i,p} + y^{t-1}_{i,p}\right)$$
$$\le \overline{K}^1_i - \sum_{\tau=1}^{t}\sum_{j\in\mathcal{S}^o} z^\tau_{i,j}, \qquad i \in \mathcal{S}^c,\ t \in \mathcal{T} \tag{8}$$

$$\sum_{p\in\mathcal{P}} \mu_{j,p}\left(b^t_{j,p} + \sum_{\ell\in\mathcal{L}\setminus\{j\}} x^t_{\ell,j,p} + y^{t-1}_{j,p}\right) \le \sum_{\tau=1}^{t}\sum_{i\in\mathcal{S}^c} z^\tau_{i,j}, \qquad j \in \mathcal{S}^o,\ t \in \mathcal{T} \tag{9}$$

$$\sum_{p\in\mathcal{P}} \mu_{\ell,p} \left(b^t_{\ell,p} + \sum_{\ell'\in\mathcal{L}\setminus\{\ell\}} x^t_{\ell',\ell,p} + y^{t-1}_{\ell,p} \right) \leq \overline{K}^t_\ell, \qquad \ell \in \mathcal{L}\setminus\mathcal{S},\ t\in\mathcal{T} \tag{10}$$

$$\sum_{p\in\mathcal{P}} \mu_{\ell,p} \left(b^t_{\ell,p} + \sum_{\ell'\in\mathcal{L}\setminus\{\ell\}} x^t_{\ell',\ell,p} + y^{t-1}_{\ell,p} \right) \geq \underline{K}^t_\ell\, \delta^t_\ell, \qquad \ell \in \mathcal{S},\ t\in\mathcal{T} \tag{11}$$

$$\delta^t_i \geq \delta^{t+1}_i, \qquad i\in\mathcal{S}^c,\ t\in\mathcal{T}\setminus\{n\} \tag{12}$$

$$\delta^t_j \leq \delta^{t+1}_j, \qquad j\in\mathcal{S}^o,\ t\in\mathcal{T}\setminus\{n\} \tag{13}$$

$$\sum_{j\in\mathcal{S}^o} FC^1_j\, \delta^2_j + \xi^1 = B^1 \tag{14}$$

$$\begin{aligned}&\sum_{i\in\mathcal{S}^c}\sum_{j\in\mathcal{S}^o} MC^t_{i,j}\, z^t_{i,j} + \sum_{i\in\mathcal{S}^c} SC^t_i \left(\delta^{t-1}_i - \delta^t_i\right)\\ &\quad + \sum_{j\in\mathcal{S}^o} FC^t_j \left(\delta^{t+1}_j - \delta^t_j\right) + \xi^t = B^t + \alpha^{t-1}\,\xi^{t-1}, \qquad t\in\mathcal{T}\setminus\{1,n\}\end{aligned} \tag{15}$$

$$\begin{aligned}&\sum_{i\in\mathcal{S}^c}\sum_{j\in\mathcal{S}^o} MC^n_{i,j}\, z^n_{i,j} + \sum_{i\in\mathcal{S}^c} SC^n_i \left(\delta^{n-1}_i - \delta^n_i\right) + \xi^n\\ &\quad = B^n + \alpha^{n-1}\,\xi^{n-1}\end{aligned} \tag{16}$$

$$b^t_{\ell,p} \geq 0, \qquad \ell\in\mathcal{L},\ p\in\mathcal{P},\ t\in\mathcal{T} \tag{17}$$

$$x^t_{\ell,\ell',p} \geq 0, \qquad \ell\in\mathcal{L},\ \ell'\in\mathcal{L}\setminus\{\ell\},\ p\in\mathcal{P},\ t\in\mathcal{T} \tag{18}$$

$$y^t_{\ell,p} \geq 0, \qquad \ell\in\mathcal{L},\ p\in\mathcal{P},\ t\in\mathcal{T} \tag{19}$$

$$z^t_{i,j} \geq 0, \qquad i\in\mathcal{S}^c,\ j\in\mathcal{S}^o,\ t\in\mathcal{T} \tag{20}$$

$$\xi^t \geq 0, \qquad t\in\mathcal{T} \tag{21}$$

$$\delta^t_\ell \in \{0,1\}, \qquad \ell\in\mathcal{S},\ t\in\mathcal{T}\,. \tag{22}$$

In the above formulation, the objective is to operate the supply chain network at minimum cost as stated by (3). The costs to minimize include production/supply costs, transportation costs between facilities, inventory holding costs, and fixed facility operating costs.

Constraints (4) are the usual flow conservation conditions which must hold for each product, facility and period. Inequalities (5)–(7) ensure that only feasible capacity relocations take place during the planning horizon. Note that constraints (5) also guarantee that only operating existing facilities can have their capacity moved to new facilities. Furthermore, constraints (6) also impose that by period t a new facility has been constructed at site j in order for a capacity relocation to take place. Constraints (7) (with ϵ a sufficiently small positive number) state that if the capacity of an existing facility has been completely transferred then the facility has to be closed. The combination of (5) and (7) ensures that if an existing facility does not operate in a given period then its entire capacity was removed in one of the previous periods. Moreover, by (7) no more capacity can be shifted out of such a facility than the one available at the beginning of the planning horizon. Constraints (8)–(10) impose that the capacity of each facility is not exceeded in each period. Observe that inequalities (8) also prevent any supply chain activities from taking place in existing facilities whose capacity has been totally relocated. Inequalities (11) state that it is only worth to operate a selectable facility if its output is above a given minimum level. Constraints (12) and (13) allow the configuration of each selectable facility to change at most once. Hence, if an existing facility is closed, it cannot be re-opened. Similarly, when a new facility is established it will remain in operation until the end of the planning horizon. Conditions (14)–(16) are budget constraints. In each period there is a limited amount of capital that can be spent on capacity transfers, shutting down existing facilities and/or setting up new facilities. This amount is given by the budget initially available in that period plus the discounted capital not invested in previous periods. In the first period, the allowed investments regard setting up new facilities that will start operating at the beginning of the second period (cf. (14)). In each one of the following periods $t \in \mathcal{T} \setminus \{1, n\}$, the available capital may cover capacity transfers, the costs incurred by closing existing facilities at the end of period $t-1$, and setups of new facilities that start operating at the beginning of period $t+1$ (cf. (15)). In the last period n, the allowed investments concern capacity transfers as well as shutdowns of facilities that ceased operating at the end of period $n-1$ (cf. (16)). Finally, constraints (17)–(22) represent non-negativity and integrality conditions.

Under special conditions, the above formulation can be simplified as stated by the next result.

Lemma 3.1 *If $\underline{K}_i^t > 0$ for every $i \in \mathcal{S}^c$ and $t \in \mathcal{T}$ then constraints (7) are redundant.*

Proof: For every $i \in \mathcal{S}^c$ and $t \in \mathcal{T}$, constraints (8) and (11) imply that the capacity operating in an existing selectable facility i in a given period t must be at least equal to the corresponding minimum throughput, that is, $\sum_{\tau=1}^{t} \sum_{j \in \mathcal{S}^o} z_{i,j}^{\tau} + \underline{K}_i^t \, \delta_i^t \leq \overline{K}_i^1$. If $\underline{K}_i^t > 0$ for $i \in \mathcal{S}^c$ and $t \in \mathcal{T}$ then the above inequalities are stronger than (7), and thus (7) can be eliminated from formulation (P). □

It should be noted that when a given minimum throughput is equal to zero, the constraint (7) associated with the corresponding facility/period combination must be explicitly considered.

It is easy to verify that formulation (P) contains

- $\mid \mathcal{T} \mid \cdot (\mid \mathcal{L} \mid \cdot \mid \mathcal{P} \mid +3 \mid \mathcal{S} \mid + \mid \mathcal{S}^c \mid + \mid \mathcal{L} \mid +1) - 3 \mid \mathcal{S} \mid$ constraints,
- $\mid \mathcal{L} \mid \cdot (\mid \mathcal{L} \mid + 1) \cdot \mid \mathcal{P} \mid \cdot \mid \mathcal{T} \mid + \mid \mathcal{T} \mid (\mid \mathcal{S}^c \mid \cdot \mid \mathcal{S}^o \mid +1) - \mid \mathcal{S}^c \mid \cdot \mid \mathcal{S}^o \mid$ continuous variables,
- and $\mid \mathcal{S} \mid \cdot (\mid \mathcal{T} \mid -1)$ binary variables.

A moderate size problem with 10 planning periods, 50 customers, 5 plants, 10 existing DCs, 20 potential sites for new DCs, and 10 different product types (i.e. $\mid \mathcal{T} \mid = 10, \mid \mathcal{L} \mid = 85$ with $\mid \mathcal{S}^c \mid = 10$, $\mid \mathcal{S}^o \mid = 20$, $\mid \mathcal{S} \mid = 30$, and $\mid \mathcal{P} \mid = 10$) leads to a formulation involving 10270 constraints, and 732810 continuous and 270 binary variables. These values are clearly upper bounds, especially on the number of variables, since in practice the transportation variables $x^t_{\ell,\ell',p}$ are not defined for every pair of facilities ℓ and ℓ', and every product p. Nevertheless, (P) is a large-scale MIP problem.

3.3 Alternative formulation

The decision variables δ^t_ℓ describing the state of the selectable facilities $\ell \in \mathcal{S}$ in each period can be found in many dynamic facility location models in the literature (e.g. Canel *et al.* [8], Chardaire *et al.* [9], and Roodman and Schwarz [34,35]). However, it is possible to develop an alternative formulation to (P) by redefining the location variables in a similar way as done by e.g. Hinojosa *et al.* [18], Saldanha da Gama [36], Van Roy and Erlenkotter [40], and Velten [41]. In Section 6 we will look closely at the benefits of such a variable redefinition. We replace the δ^t_ℓ variables by the following set:

$$\eta^t_i = \begin{cases} 1 & \text{if the existing facility } i \in \mathcal{S}^c \text{ is closed at the end of period } t \in \mathcal{T} \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

$$\eta^t_j = \begin{cases} 1 & \text{if a new facility starts operating at site } j \in \mathcal{S}^o \text{ at the beginning} \\ & \text{of period } t \in \mathcal{T} \\ 0 & \text{otherwise} \end{cases} \tag{24}$$

It should be noted that it is allowed neither to close a facility in the last period nor to have a new facility operating in the first period. The following relations are straightforward.

a) *Existing (selectable) facilities, $i \in S^c$:*

- $\eta_i^1 = \delta_i^1 - \delta_i^2 = 1 - \delta_i^2$
- $\eta_i^t = \delta_i^t - \delta_i^{t+1}, \quad t \in \mathcal{T} \setminus \{1, n\}$
- $\eta_i^n = 0$

b) *New facilities, $j \in S^o$:*

- $\eta_j^1 = 0$
- $\eta_j^2 = \delta_j^2 - \delta_j^1 = \delta_j^2$
- $\eta_j^t = \delta_j^t - \delta_j^{t-1}, \quad t \in \mathcal{T} \setminus \{1, 2\}$

Consequently,

$$\delta_i^t = 1 - \sum_{\tau=1}^{t-1} \eta_i^\tau, \quad i \in \mathcal{S}^c,\, t \in \mathcal{T}, \tag{25}$$

and

$$\delta_j^t = \sum_{\tau=1}^{t} \eta_j^\tau, \qquad j \in \mathcal{S}^o,\, t \in \mathcal{T}\,. \tag{26}$$

An alternative MIP formulation to (P) is obtained by using the above relations (25) and (26) in (3), (5)–(7), (11), and (14)–(16). The facility configuration constraints (12) and (13) are replaced by the following set:

$$\sum_{t \in \mathcal{T}} \eta_\ell^t \leq 1, \quad \ell \in \mathcal{S} = \mathcal{S}^c \cup \mathcal{S}^o\,. \tag{27}$$

For $\ell \in \mathcal{S}^c$, inequalities (27) state that once an existing facility is closed it cannot be re-opened, while for $\ell \in \mathcal{S}^o$ the above inequalities ensure that if a new facility is established then it will remain in operation until the end of the planning horizon. Constraints (4), (8)–(10), and (17)–(21) of (P) remain unchanged. Furthermore, the integrality conditions (22) are replaced by

$$\eta_i^t,\, \eta_j^t \in \{0, 1\}, \quad i \in \mathcal{S}^c,\, j \in \mathcal{S}^o,\, t \in \mathcal{T}\,. \tag{22'}$$

The new formulation, denoted by (AP), has the same number of variables as (P). However, due to (27), (AP) has $\mid \mathcal{S} \mid \cdot (\mid \mathcal{T} \mid - 2)$ less constraints (for $T > 2$). Moreover, the two models are equivalent, which is a direct consequence of the relations between the two different types of location variables as described by (25) and (26). For certain dynamic facility location problems, Hinojosa *et al.* [18], Saldanha da Gama [36], Van Roy and Erlenkotter [40], and Velten [41] showed that it is advantageous to use the new variables. Hinojosa *et al.* [18] developed a Lagrangean relaxation for a dynamic, two-level, multi-commodity location problem. The procedure was improved and extended by Velten [41] to include inventory decisions. Using standard mathematical programming software, Saldanha da Gama [36] showed that with the new variables it is much less time consuming to obtain the optimal solution or the lower bound provided by the linear relaxation of the simple dynamic location problem with budget constraints of the type of (14)–(16). Van Roy and Erlenkotter [40] took advantage of the new variables to develop a dual-based procedure for solving the simple dynamic location problem. In Section 6 our computational experience will also indicate a preference for using the alternative model over (P) for certain network configurations.

4 Extensions

To cope with changing demand patterns, two extensions of the base model (P) are introduced in this section, namely, the capacity expansion (see Section 4.1) and capacity reduction scenarios (see Section 4.2). Furthermore, the special case of modular capacity transfers will be discussed in Section 4.3. To ease the exposition, we show in Appendix A that our base model includes many well known dynamic facility location problems in which network design decisions are restricted to opening new facilities and closing existing ones. The reduction to a simple dynamic location problem makes use of elements of the extensions described in this section.

4.1 Capacity expansion

When a company anticipates a growth in its sales volumes (e.g., based upon customer demand forecasts), it needs to adopt additional restructuring measures. This entails expanding the supply chain network to respond to increasing demand patterns. Expansion plans may result in extending the capacity of existing facilities and/or establishing additional facilities. In our base model (P), the total capacity available at the beginning of the planning project does not change over the time horizon. However, it is relatively easy to extend (P) to an expansion scenario.

We consider a *fictitious facility*, denoted by i_0, which concentrates the total additional capacity, $\overline{K}_{i_0}^1$, required to cope with increasing demands. This facility has a similar status to that of selectable facilities in $\mathcal{S}^c$, although it cannot satisfy any demand requirements. Its capacity can be shifted to both existing and new facilities. As a result, either an existing facility will have its capacity expanded or it will be (partly) relocated to one or more new sites. Both operations are not permitted. Regarding the new facilities, they can receive capacity not only from existing (selectable) facilities, but also from the fictitious facility i_0. As in the base model, new facilities cannot be downgraded.

Observe that the capacity variations described in (1) for each facility $i \in \mathcal{S}^c$ do not permit the expansion of existing selectable facilities. Therefore, condition (1) is now dropped for at least one facility in the set $\mathcal{S}^c$. With respect to the potential sites for new facilities, assumptions (2) also hold here.

Under the new problem settings, the following decision variables are required in addition to those defined in Section 3.1.

Additional decision variables

$z_{i_0,\ell}^t$ = amount of capacity shifted from the fictitious facility i_0 to a facility $\ell \in \mathcal{S}$ at the beginning of period $t \in \mathcal{T}$

$$\rho_i = \begin{cases} 1 & \text{if the existing facility } i \in \mathcal{S}^c \text{ has its capacity extended during the planning horizon} \\ 0 & \text{otherwise} \end{cases}$$

Regarding the moving costs, we extend them to the new situation by defining $MC_{i_0,\ell}^t$ as the unit variable capacity acquisition cost in facility $\ell \in \mathcal{S}$ and period $t \in \mathcal{T}$. No other costs are charged to the fictitious facility i_0. At the end of Section 2.2 we mentioned the reasons why capacity transfers were not allowed in the first period. However, since for existing selectable facilities an infrastructure already exists at the beginning of the planning horizon, we will relax this assumption and also permit the expansion of existing facilities in the first period. In other words, capacity shifts from the fictitious facility i_0 to any facility $i \in \mathcal{S}^c$ are allowed in $t = 1$, i.e., $z_{i_0,i}^1 \geq 0$. With respect to new facilities, we keep the initial assumption that $z_{i_0,j}^1 = 0$ for every $j \in \mathcal{S}^o$.

The new MIP formulation has the same objective function (3)as the base model. Regarding the constraints, the following modifications are required. The term $\sum_{\tau=1}^{t} z_{i_0,i}^{\tau}$ is added to the left-hand side of constraints (5), and to the right-hand side of constraints (8). Furthermore, the term $\sum_{i\in\mathcal{S}^c} z^{\tau}_{i,j}$ in constraints (6) and (9) is replaced by $\sum_{i\in\mathcal{S}^c\cup\{i_0\}} z^{\tau}_{i,j}$. Regarding the budget constraints, we need to add the moving costs $\sum_{i\in\mathcal{S}^c} MC^1_{i_0,i}\, z^1_{i_0,i}$ to the left-hand side of (14). In equations (15) and (16), the term describing the capacity moving costs is replaced by $\sum_{i\in\mathcal{S}^c\cup\{i_0\}}\sum_{j\in\mathcal{S}^o} MC^t_{i,j}\, z^t_{i,j} + \sum_{i\in\mathcal{S}^c} MC^t_{i_0,i}\, z^t_{i_0,i}$ for every $t\in\mathcal{T}\setminus\{1\}$. In addition to constraints (4), (7), (10)–(14), and (17)-(22), the following new conditions are required.

$$\rho_i \le \delta^n_i, \qquad i\in\mathcal{S}^c \tag{28}$$

$$\sum_{t\in\mathcal{T}} z^t_{i_0,i} \le \overline{K}^1_{i_0}\,\rho_i, \qquad i\in\mathcal{S}^c \tag{29}$$

$$\sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{S}^o} z^t_{i,j} \le \overline{K}^1_i\,(1-\rho_i), \qquad i\in\mathcal{S}^c \tag{30}$$

$$\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{S}^c} z^t_{i_0,i} + \sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{S}^o} z^t_{i_0,j} \le \overline{K}^1_{i_0}, \tag{31}$$

$$z^t_{i_0,\ell} \ge 0, \qquad \ell\in\mathcal{S}, t\in\mathcal{T} \tag{32}$$

$$\rho_i \in \{0,1\}, \qquad i\in\mathcal{S}^c \tag{33}$$

Constraints (28) together with the configuration constraints (12) guarantee that if an existing selectable facility $i\in\mathcal{S}^c$ is expanded then it will remain in operation during the whole planning horizon. The opposite also holds, that is, in the case that facility $i\in\mathcal{S}^c$ is not operated in the last period n, this means that it must have been closed in one of the previous periods and therefore could not have its capacity expanded. Constraints (29) together with (30) ensure that an existing facility can either have its capacity expanded or relocated to new sites, but both operations cannot take place during the planning horizon. Observe that if the size of an existing selectable facility is not increased (i.e., $\rho_i = 0$, $i\in\mathcal{S}^c$), then by (29) any capacity transfers from the fictitious facility i_0 are prevented. If capacity is extended (i.e., $\rho_i = 1$, $i\in\mathcal{S}^c$) then by (30) no part of it can be later transferred. Constraint (31) states that the capacity of the fictitious facility cannot be exceeded.

Compared to the base model (P), the extended MIP formulation has $3\mid\mathcal{S}^c\mid+1$ new constraints, $\mid\mathcal{S}\mid\cdot\mid\mathcal{T}\mid-\mid\mathcal{S}^o\mid$ new continuous variables, and $\mid\mathcal{S}^c\mid$ new binary variables.

We conclude this section by remarking that, if desired, capacity expansions can easily be imposed by varying the capacity of the fictitious facility according to (1), and replacing constraint (31) by the appropriate conditions.

4.2 Capacity reduction

As opposed to the situation discussed in the previous section, we now address a capacity reduction scenario. Products approaching the end of their life cycles are faced with declining demand patterns, and as a result, capacity that was initially reserved for them becomes redundant. Also during periods of economic downturns, demand patterns tend to be very volatile and excess capacities need to be reduced.

Again we consider a *fictitious facility*, denoted by j_0, which can be established with all excess capacity from existing facilities. This implies that j_0 has a similar status to that of the potential new facilities in $\mathcal{S}^o$, but cannot satisfy any demand requirements. We introduce an upper bound $\overline{K}_{j_0}^t$ on the overall capacity that can be reduced until period $t \in \mathcal{T}$. Condition (2) may be also imposed here. Under these new problem settings, the following decision variables are required in addition to those defined in Section 3.1.

Additional decision variables

z_{i,j_0}^t = amount of capacity shifted from an existing facility $i \in \mathcal{S}^c$ to the fictitious facility j_0, at the beginning of period $t \in \mathcal{T}$

$$\delta_{j_0}^t = \begin{cases} 1 & \text{if facility } j_0 \in \mathcal{S} \text{ is operated during period } t \in \mathcal{T} \\ 0 & \text{otherwise} \end{cases}$$

Analogous to the capacity expansion case, we will allow existing facilities to have their capacities decreased already in the first period, i.e., $z_{i,j_0}^1 \geq 0$ for $i \in \mathcal{S}^c$. The required changes in the base model (P) correspond to replacing $\mathcal{S}^o$ by $\mathcal{S}^o \cup \{j_0\}$ in constraints (5)–(8), (13), (15), (16) and (20). Moreover, the capacity moving costs $\sum_{i \in \mathcal{S}^c} MC_{i,j_0}^1 z_{i,j_0}^1$ are added to the left-hand side of the budget constraint (14) for the first period. The integrality conditions $\delta_{j_0}^t \in \{0,1\}$ for every $t \in \mathcal{T}$ are also added. As a result, the extended formulation has $2 \mid \mathcal{T} \mid -1$ constraints, $\mid \mathcal{S}^c \mid \cdot \mid \mathcal{T} \mid$ continuous variables, and $\mid \mathcal{T} \mid$ binary variables in addition to those in model (P). Finally, we observe that, if desired, it is easy to impose a minimum capacity reduction level per period by defining a lower bound $\underline{K}_{j_0}^t$ for every $t \in \mathcal{T}$, and adding the constraints $\sum_{\tau=1}^{t} \sum_{i \in \mathcal{S}^c} z_{i,j_0}^{\tau} \geq \underline{K}_{j_0}^t \, \delta_{j_0}^t$, $t \in \mathcal{T}$.

It should be noted that in many situations the fixed setup costs $FC_{j_0}^t$ and the operating costs $OC_{j_0}^t$ of the fictitious facility may be zero, while the unit variable costs of moving capacity to this facility (MC_{i,j_0}^t, $i \in \mathcal{S}^c$) may be negative. This corresponds, for instance, to the situation where a return is obtained by selling infrastructures. In this case, reducing capacity is profitable. If the capacity reduction costs are positive then two scenarios may still lead to a decrease of the available capacities. The first has to do with the maximum allowed capacities. If, in some period, the overall sum of these capacities is below the total initial capacity then, whatever the capacity reduction costs, capacity reduction will necessarily occur. The second scenario regards the magnitude of the costs. It may be worth reducing the capacity when its cost is lower than the cost one would have to pay for having that capacity operating. In fact, due to the minimum throughputs, the total cost for having a certain capacity operating may be high.

4.3 Modular capacity transfers

Until now we modeled capacity relocation through the continuous variables $z_{i,j}^t$, $i \in \mathcal{S}^c$, $j \in \mathcal{S}^o$, $t \in \mathcal{T}$. This is a good approximation for many applications in which the size of the capacity shifts is large. However, there are cases in which the possible transfer sizes are restricted to discrete amounts. Applications can be found in telecommunications and production settings, where the facilities (e.g., various types of concentrators, production equipment) are manufactured in a limited set of sizes. In addition, the modular case also permits to model economies of scale in the costs of setting up new facilities and relocating existing facilities. Hence, in this section we focus on modular capacity transfers. Newly established facilities obtain capacity by adding modules chosen from those available at existing locations. Let $\mathcal{M}$ denote the set of module types and let us define S_m as the size of a module of type $m \in \mathcal{M}$. We assume that $\mathcal{S}_m < \mathcal{S}_{m+1}$ for every $m \in \mathcal{M} \setminus \{M\}$, with M denoting the largest module. Hence, the size of an existing facility $i \in \mathcal{S}^c$ at the beginning of the planning horizon (i.e. $\overline{K}_i^1$) is given by the sum of the sizes of the modules that are operating in that facility. Under these new problem settings, the following decision variables are required in addition to those already defined in Section 3.1.

Additional decision variables

$$\gamma_{i,j,m}^t \quad = \quad \begin{cases} 1 & \text{if a module of type } m \in \mathcal{M} \text{ is shifted from facility } i \in \mathcal{S}^c \text{ to} \\ & \text{facility } j \in \mathcal{S}^o \text{ in period } t \in \mathcal{T} \\ 0 & \text{otherwise} \end{cases}$$

To extend our base model (P) to the new scenario, we transform the continuous decision variables $z_{i,j}^t$, which represent the capacity shifts, into discrete variables by setting

$$z_{i,j}^t = \sum_{m \in \mathcal{M}} \mathcal{S}_m \, \gamma_{i,j,m}^t, \qquad\qquad i \in \mathcal{S}^c,\ j \in \mathcal{S}^o,\ t \in \mathcal{T}\ .$$

Naturally, in the first period $\gamma^1_{i,j,m} = 0$, for $i \in \mathcal{S}^c$, $j \in \mathcal{S}^o$ and $m \in \mathcal{M}$. Regarding the costs of shifting capacity between facilities, we introduce $MC^t_{i,j,m}$ as the fixed cost of moving a module of type $m \in \mathcal{M}$ from the existing facility $i \in \mathcal{S}^c$ to a new facility established at site $j \in \mathcal{S}^o$ at the beginning of period $t \in \mathcal{T} \setminus \{1\}$. Clearly, the budget constraints (15) and (16) are affected by this cost redefinition. The term describing the capacity moving costs is replaced by $\sum_{i \in \mathcal{S}^c} \sum_{j \in \mathcal{S}^o} \sum_{m \in \mathcal{M}} MC^t_{i,j,m}\, \gamma^t_{i,j,m}$ for every $t \in \mathcal{T} \setminus \{1\}$.

Finally, the extended model has the same number of constraints as (P), but $\mid \mathcal{S}^c \mid \cdot \mid \mathcal{S}^o \mid \cdot \mid \mathcal{M} \mid \cdot (\mid \mathcal{T} \mid -1)$ additional binary variables. Regarding the continuous variables, the original total number is reduced by $\mid \mathcal{S}^c \mid \cdot \mid \mathcal{S}^o \mid \cdot (\mid \mathcal{T} \mid -1)$.

5 Relation with existing literature

The modeling framework presented in Sections 3 and 4 generalizes many of the models proposed in the literature (see also Appendix A). Table 1 presents a classification of facility location problems. The table is not intended to provide an exhaustive review but rather to illustrate in which way this generalization occurs. Each column of Table 1 represents one of nine criteria: type of planning horizon, type of objective function, number of commodities, number of facility levels in addition to the customer level, number of echelons to be located, consideration of relocation and inventory decisions, and finally, inclusion of capacity and budget constraints. As far as capacity constraints are concerned, one finds essentially three situations: the usual type of constraints, the possibility of expanding or reducing operating capacity, and modular capacities. For each analytical study in Table 1 which explicitly considers capacity constraints, we detail the corresponding case(s).

Observing Table 1, one realizes that none of the existing models deals simultaneously with all the aspects considered in Sections 3 and 4. In particular, three elements are scarcely focused together in the literature, namely facility relocation, inventory opportunities and budget constraints.

Due to the fact that our modeling framework extends many existing models (cf. Appendix A), solution procedures proposed in the papers listed in Table 1 seem a reasonable starting point for developing analytical approaches to solve the new enlarged models. Even though the paper maintains a modeling rather than an algorithmic focus, we summarize next the approaches that have been developed to deal with simpler models.

The use of commercially available mathematical programming software was often con-

Table 1: Relation of model (P) with existing models

	Planning horizon (S/D)[1]	Objective function (C/P/M)[2]	Multi-commodity	Facility levels[3]	Location levels (A/I/M)[4]	Relocation	Inventory	Capacity const. (C/E/R/M)[5]	Budget constraints
Model (P) and extensions	D	C	√	M	M	√	√	C,E,R,M	√
Antunes and Peeters [1]	D	C	-	1	A	-	-	E,R,M	√
Ballou [2]	D	C	-	1	A	√	-	-	-
Barros [3], Barros and Labbé [4]	S	P	-	2	A	-	-	-	-
Canel and Khumawala [7]	D	C	-	1	A	-	-	-	-
Canel *et al.* [8]	D	C	√	2	I	-	-	C	-
Chardaire *et al.* [9]	D	C	-	1	A	-	-	-	-
Elson [10]	S	C	√	2	I	-	-	C	-
Erlenkotter [11]	D	C	-	1	A	-	-	E	-
Fong and Srinivasan [13, 14]	D	C	-	1	A	-	-	E	-
Galvão and Santibañez-Gonzalez [15]	D	C	-	1	A	-	-	-	-
Geoffrion and Graves [16]	S	C	√	2	I	-	-	C	-
Gue [17]	D	C	√	1	A	-	√	C	-
Hinojosa *et al.* [18]	D	C	√	2	A	-	-	C	-
Hormozi and Khumawala [20]	D	C	-	2	I	-	-	C	-
Jacobsen [23]	D	C	-	1	A	-	-	E	-
Kaufman *et al.* [26]	S	C	-	2	A	-	-	-	-
Kelly and Marucheck [27]	D	C	-	1	A	-	-	C	-
Khumawala and Whybark [28]	D	C	-	1	A	-	-	-	-
Lee and Luss [29]	D	C	-	1	A	√	-	E	-
Melachrinoudis and Min [30, 32]	D	M	-	2	I	√	-	E,R	√
Melachrinoudis *et al.* [31]	D	M	-	1	A	-	-	C	√
Pirkul and Jayaraman [33]	S	C	√	1	A	-	-	C	-
Roodman and Schwarz [34, 35]	D	C	-	1	A	-	-	-	-
Saldanha da Gama [36]	D	C	-	1	A	-	-	C	√
Shulman [37]	D	C	-	1	A	-	-	E,M	-
Sweeney and Tatham [38]	D	C	-	2	I	-	-	C	-
Tcha and Lee [39]	S	P	-	M	M	-	-	-	-
Van Roy and Erlenkotter [40]	D	C	-	1	A	-	-	-	-
Velten [41]	D	C	√	2	A	-	√	C	-
Wang *et al.* [42]	S	C	-	1	A	-	-	-	√

1 - S:Static, D:Dynamic
2 - C:Cost, P:Profit, M:Multiple
3 - M:Multiple (no restrictions on the number of facility levels)
4 - A:All, I:Intermediate, M:Multiple (no restrictions on the number of location levels)
5 - C:Capacitated, E:capacity Expansion, R:capacity Reduction, M:Modular capacities

sidered namely, by Elson [10], Gue [17], Melachrinoudis and Min [30], Melachrinoudis *et al.* [31], Min and Melachrinoudis [32], and Saldanha da Gama [36]. In order to explore specific characteristics of the problems, tailored branch-and-bound procedures have been developed. This was the approach followed by Barros [3], Barros and Labbé [4], Canel and Khumawala [7], Canel *et al.* [8], Kaufman *et al.* [26], Khumawala and Whybark [28], Roodman and Schwarz [34, 35], Tcha and Lee [39], and Van Roy and Erlenkotter [40]. Other exact procedures based upon Benders decomposition were proposed by Geoffrion and Graves [16] and Kelly and Marucheck [27]. Finally, dynamic programming approaches were considered by Lee and Luss [29], Saldanha da Gama [36], and Sweeney and Tatham [38]. Hormozi and Khumawala [20] combined mixed integer with dynamic programming.

Due to the complexity of the problems, much attention has been focused on obtaining lower and/or upper bounds. Regarding lower bounds, heuristics for the dual of a linear relaxation were developed by Chardaire *et al.* [9], Saldanha da Gama [36], and Van Roy and Erlenkotter [40]. Lagrangean relaxation was used by Galvão and Santibañez-Gonzalez [15], Hinojosa *et al.* [18], Pirkul and Jayaraman [33], Saldanha da Gama [36], Shulman [37], and Velten [41]. In all these papers, a Lagrangean heuristic was developed to obtain a feasible solution to the problem. Finally, meta-heuristics, namely simulated annealing, were proposed by Antunes and Peeters [1] and Chardaire *et al.* [9] who also analyzed ADD/DROP/exchange heuristics. This last type of procedures was also proposed by Erlenkotter [11], Fong and Srinivasan [13,14], Roodman and Schwarz [34,35] and Saldanha da Gama [36]. Lagrangean, greedy and tabu search based heuristics were developed by Wang *et al.* [42]. Jacobsen [23] proposed two heuristic procedures based upon the determination of a shortest path in a specific network.

6 Computational experience

In this section we present the results of our numerical tests. The MIP formulations described in the previous sections were implemented using the modeling language ILOG OPL Studio 3.6 [22], and a variety of test problems were solved with standard mathematical programming software, namely with the branch-and-bound algorithm of ILOG CPLEX 8.0 [21], on a Pentium III PC with 2.6 GHz processor and 2 GB RAM. Our computational experiments were guided by three objectives:

- to identify the most effective formulation between the two different models described in Sections 3.2 and 3.3, and to measure the quality of the lower bounds produced by their linear relaxations;

- to analyze the impact of gradually relaxing certain assumptions in the original model such as excluding budget restrictions on investments, and reducing the decision space by removing facility relocation decisions and thus coming closer to location models with exclusively opening and closing decisions;
- to analyze the impact of various problem characteristics such as the size and configuration of the networks, and the number of periods, products and customers.

We randomly generated three classes of problems corresponding to different network configurations as depicted in Figure 3. The generated problems vary from single to tri-

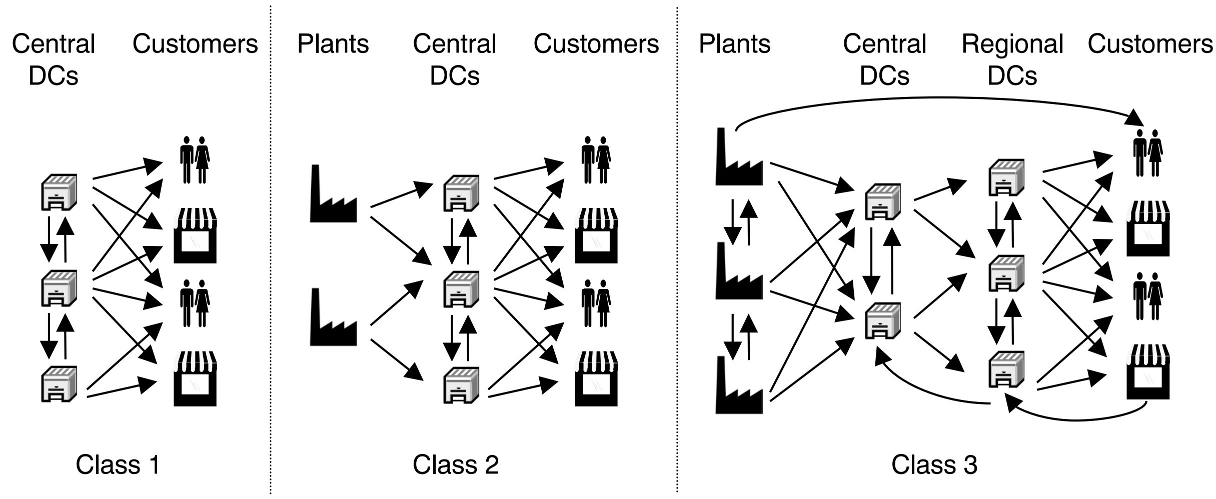

Figure 3: The three classes of network configurations generated.

echelon networks with various alternatives for the flow of products such as inter-facility transportation, direct deliveries from the plant to the customer level, and reverse arcs for the return of goods. Details regarding the characteristics of the 95 generated problem instances can be found in Appendix B. In all the problems the relocation decisions involve the distribution centers. Furthermore, all data (costs, capacities, demands, etc.) was drawn at random from a uniform distribution over given intervals. As a result, the transportation costs are neither based on distances between facilities nor satisfy the triangle inequality.

This aspect contributes to an increase in the difficulty of solving the generated problem instances.

The first experiments aimed at determining the most effective formulation for capacity relocation. Table 2 gives an overview of the number of variables and constraints in formulations (P) and (AP). As expected, the size of the models is largely influenced by the bulk of variables which in turn is dominated by the number of transportation arcs (i.e., variables $x^t_{\ell,\ell',p}$). Regarding the constraints, and as mentioned in Section 3.3, small savings are made with the alternative formulation.

Problem	# Variables in (P) and (AP)			# Constraints in (P)			# Constraints in (AP)		
class	aver.	min.	max.	aver.	min.	max.	aver.	min.	max.
1	33385.1	15943	55924	3231.9	1691	4998	3201.9	1691	4968
2	40504.0	22684	58644	3824.0	2374	5174	3794.0	2344	5144
3	32268.8	24168	40440	2424.8	1833	3111	2400.8	1833	3047

Table 2: Size of the problem instances.

Table 3 displays the average, minimum and maximum CPU times (in seconds) required to attain optimality in each problem class for various variants of the original model. The relative percentage deviation ("LP-gap") between the optimal solution value and the lower bound given by the linear relaxation serves as a measure of tightness of the MIP formulations. The first two blocks of rows in Table 3 refer to formulations (P) and (AP). It can be seen that all problems could be solved optimally in less than 40 minutes. Although the linear relaxations of both models are equally tight, the computation time required to solve the various instances varies from class to class. At first sight it may be peculiar that the problems in the first class are more difficult to solve than those in the second class. Although the networks of the problems in the latter class have an additional facility level, the plant level, the number of plants is considerably smaller than the number of DCs (the location level). Since not every plant is linked to every DC for each product, the choice to operate a given DC is not so difficult to take as in class 1. Consequently, the flow of products from the DCs to the customers is further restricted by a subset of the existing transportation arcs. This leads to a general decrease in the total computational effort. Nevertheless, the alternative formulation seems to be more efficient for class 2. This observation is in part supported by Hinojosa *et al.* [18], Saldanha da Gama [36] and Velten [41] who developed formulations such as (AP) for problems where the underlying network configuration is similar to that of class 2 (excluding the inter-facility arcs). Compared to the other classes, problem class 3, which has more facility levels and a wide variety of transportation arcs (including some reverse arcs), requires considerably more computational effort with formulation (AP) than with (P). A close analysis of the CPU times in this class reveals that 17 out of 20 problem instances were solved in less than 10 minutes. Hence, only three problem instances (i.e. 15%) with CPU times varying between 1364.36 s and 2358.64 s are responsible for the large average given in Table 3.

	Problem	CPU time (s)			LP-gap (%)		
Formulation	class	aver.	min.	max.	aver.	min.	max.
Original (P)	1	829.60	17.67	7272.53	1.27	0.01	4.76
	2	341.66	29.14	1057.70	0.33	0.00	1.41
	3	283.78	24.89	1533.79	0.92	0.40	1.64
Alternative (AP)	1	667.83	20.94	7194.67			
	2	241.75	34.73	580.64	same as (P)		
	3	439.67	24.48	2358.64			
No bgt. const. $(AP-B)$	1	108.24	6.13	826.01	0.95	0.00	3.82
	2	48.30	14.58	272.71	0.30	0.00	1.35
	3	104.96	12.23	426.96	2.24	1.57	2.79
No relocation $(AP-R)$	1	256.09	10.92	1909.25	3.22	0.15	15.55
	2	114.31	14.27	473.15	3.50	0.11	18.62
	3	1178.33	24.72	15654.62	1.88	1.01	2.59
No relocation,	1	158.95	16.63	502.00	4.01	1.58	6.79
no bgt. const. $(AP-RB)$	2	307.34	33.39	3790.13	1.30	0.01	3.08
	3	124.70	15.50	429.46	19.45	8.69	89.82

Table 3: Performance of the formulations for various model variations.

The last three blocks of rows in Table 3 show the results obtained with simplified versions of the relocation model. We analyzed the effect of removing the budget constraints (14)–(16) from the formulation. In this case, the fixed facility opening and closing costs along with the variable capacity relocation costs are included in the objective function (3). We denote this variant of our model by $(AP - B)$ (with AP indicating that the changes were made in the alternative formulation). Furthermore, using the transformations described in Appendix A, which are based on the extensions presented in Section 4, facility relocation decisions were also excluded from the decision space. In the new model, denoted by $(AP - R)$, new facilities can only be opened at its full capacity in one single period, while existing facilities can only be closed by having their capacities reduced to zero at once. Finally, we further simplified the "pure" opening/closing variant by excluding the budget constraints. The results obtained are shown in Table 3 under $(AP - RB)$.

We observe that having a given capital for investing on facility location and relocation has a large impact on the time required to attain optimality. The exclusion of budget constraints leads to much easier problems independent of the network configuration and problem size. This is not surprising since in some of the test problems the budget constraints are occasionally binding. However, the quality of the lower bounds of the linear relaxation deteriorates in class 3. It is also interesting to observe the effect of using our model without capacity relocation decisions. While the CPU times in the first two classes clearly decrease compared to the models (P) and (AP), the impact on class 3 is considerable. A closer look to this class showed that one single instance required considerable more computational effort (15654.62 s $\approx$ 4.3 h) than the others. By removing this instance from the class, the CPU times are closer to the results obtained with (AP) (on average 416.42 s and a maximum of 2239.68 s). In this instance, and using the alternative formulation (AP), many capacity relocations take place to three new regional DCs without closing any existing DCs since the relocation costs are relatively low and the available budget is large. This strategy avoids paying fixed costs for shutting down facilities. In model $(AP-R)$ only two new regional DCs are established and several existing DCs (both regional and central) are closed. It seems that in this case the abrupt decision to close and open DCs is harder to take, while the option of gradually moving capacity between facilities is easier to manage. This observation seems to be confirmed by the results obtained with the variant $(AP - RB)$. The experiments conducted with this simplified model yielded both larger CPU times and LP-gaps than those with $(AP - B)$. When budget limitations are disregarded, adding the facility costs (opening, closing and capacity relocation costs) to the objective function leads to the search for the configuration with least total cost. Usually, less combinations with respect to changes in the status of each facility need to be assessed to find the best configuration, while more combinations for investments are considered especially when the available budget is not too tight, thus increasing the overall computational effort. Concerning the LP-relaxation, the quality of the lower bounds clearly decreases with the models restricted to opening and closing facility decisions (i.e. $(AP - R)$ and $(AP - RB)$).

The next experiments aimed at analyzing the impact of the number of periods, products and customers on the solution times when using each of the five model variants described above. The results obtained are summarized in Figures 4, 5 and 6. Each bar corresponds to the average CPU time over five problem instances. As expected, the length of the time horizon has the largest effect as shown in Figure 4 for selected problems of class 1 (single echelon networks). This is particularly striking for the formulations (P) and (AP) where the computational effort almost explodes when the planning horizon increases from 4 to 6 periods. This is caused by a large increase in the number of transportation variables.

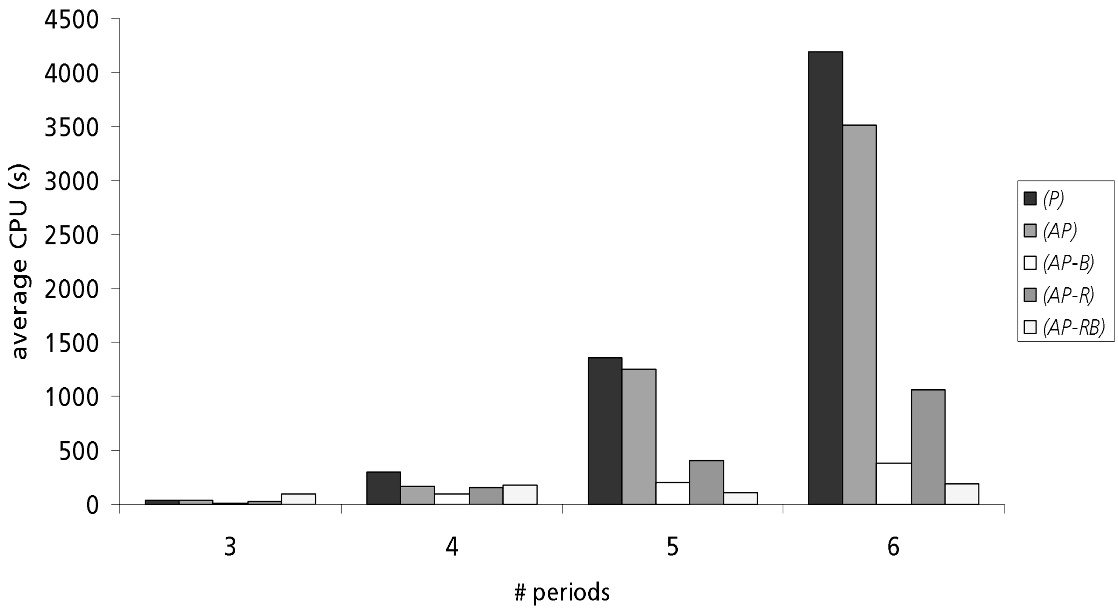

Figure 4: Effect of increasing the number of periods.

The impact of the number of products was explored using three problems of class 1. As expected, a gradual increase in the CPU time is observed. However, we note from Figure 5 that the model without facility relocation and budget constraints $(AP - RB)$ is the most affected by this change. Increasing the number of products by five adds $5 \cdot \mid \mathcal{T} \mid \cdot \mid \mathcal{L} \mid$ new constraints to the formulations (P) and (AP), while an additional period to the time horizon corresponds to adding a much larger number of constraints, namely $\mid \mathcal{L} \mid \cdot (\mid \mathcal{P} \mid + 1) + 2 \cdot \mid \mathcal{S} \mid + \mid \mathcal{S}^o \mid + 1$ new constraints.

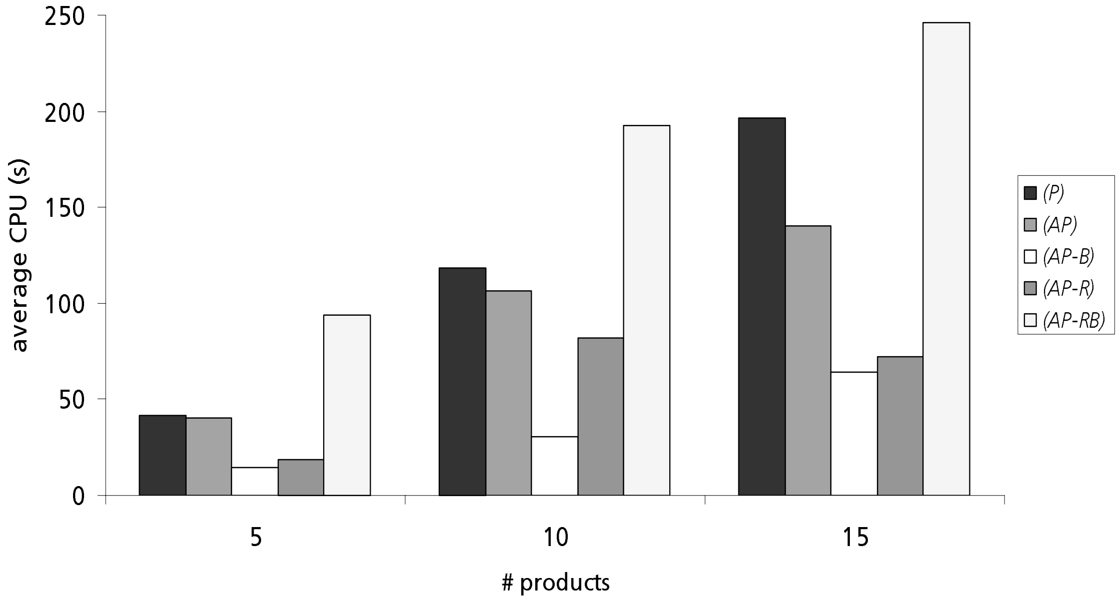

Figure 5: Effect of increasing the number of products.

Finally, using four problems of class 2 (two-echelon networks) we studied the effect of increasing the number of customers. Figure 6 shows that the problems are easier to solve when the networks grow with respect to the demand market. This is not surprising, since all other factors remain unchanged, in particular the capacities of the existing facilities. Hence, increasing the demand requirements induces larger product flows, and as a result a higher utilization of the DCs. Since almost every facility is needed to serve (directly or indirectly) customer demands, the decisions on which facilities to operate are easier to take. In other words, the number of possible facility configurations becomes smaller when the number of customers increases, and consequently the problems can be solved in less time.

7 Conclusions and further research

We described a mathematical modeling framework for dynamic facility location that captures important features of strategic supply chain planning problems. The aspects consid-

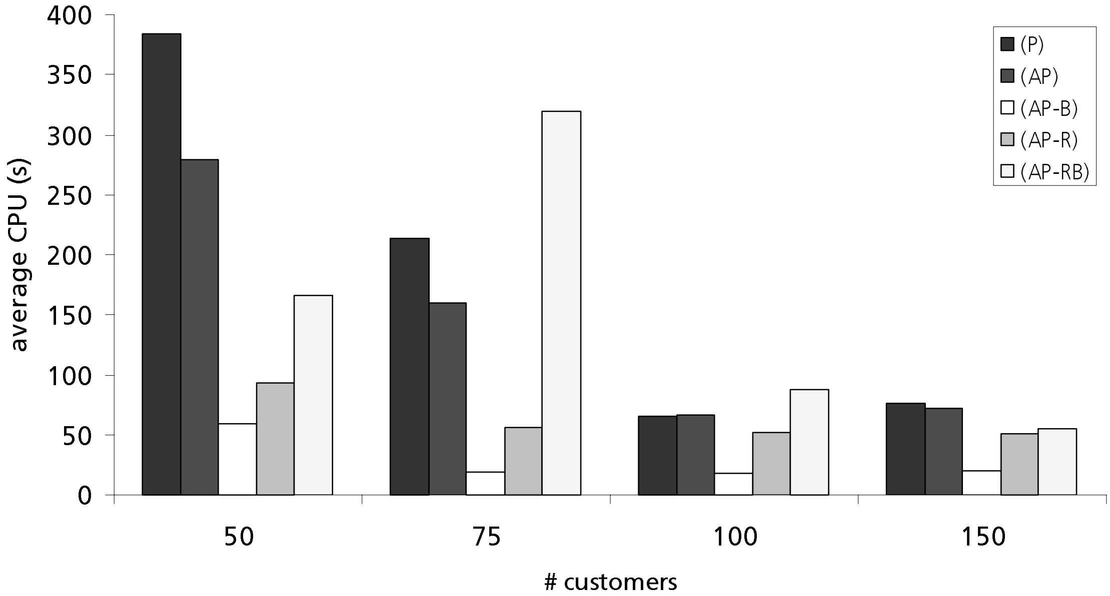

Figure 6: Effect of increasing the number of customers.

ered include the relocation of existing facilities through capacity transfers to new locations, integration of inventory, transportation and supply decisions, the availability of a given budget for investments in facility location and relocation, and the generic structure of the supply chain network. We have shown that capacity expansion and reduction scenarios as well as modular capacity shifts can easily be incorporated into our model, thus extending its already broad scope. Furthermore, an extensive comparison to other models that appeared in the literature has shown that our framework generalizes many aspects that have been considered individually. Using standard mathematical software, we solved to optimality a number of randomly generated test problems in less than 40 minutes. Moreover, we analyzed the impact of gradually relaxing certain assumptions, thus bringing the model closer to well known dynamic facility location problems. We observed that the quality of the lower bound provided by the linear relaxation deteriorates when budget limitations and capacity relocation decisions are disregarded. Sensitivity analysis was also conducted on a number of individual problem components.

The problem investigated in this paper was motivated by our work in planning and designing supply chain networks for a variety of industrial projects. Despite its practical nature, it is a relatively unstudied problem for which efficient solution methods need to be developed. Promising approaches include decomposition methods and metaheuristics such as variable neighborhood search.

In the problem addressed in this paper, fixed costs for opening and closing facilities are charged in the period prior to or following a facility configuration change. This aspect could easily be relaxed to include the payment of such costs in several consecutive periods and thus strengthening the model. In addition, economies of scale arising in transportation could also be considered. Another important extension would be to relax the assumption of determinitisc demand, costs and other factors in the problem and treat them as stochastic. However, such modifications would certainly have a strong impact on the complexity of the problem.

Clearly, the above variations and extensions of the problem will provide additional challenges to develop efficient solution methods to this very realistic and strategically significant practical problem.

Acknowledgements

Special thanks are due to Sven-Hendrik Magotsch for his help in generating the random problems and implementing the various MIP formulations with the modeling language ILOG OPL Studio.

Appendix A: Reduction to well known location problems

As mentioned in Section 1 (see also Table 1 in Section 5), in a vast majority of the dynamic location literature, network design decisions are restricted to opening new facilities. Some authors also address the possibility of closing existing facilities (e.g. Van Roy and Erlenkotter [40]). Melachrinoudis and Min [30] and Min and Melachrinoudis [32] are among the few authors who explicitly incorporate relocation decisions in their models. By making use of elements of the extensions introduced in Sections 4.1–4.3, we will show next that our general modeling framework also includes *pure* opening and closing facility scenarios. Furthermore, we will describe how to transform the base model into a simple dynamic location problem.

When facility relocation is excluded from the decision space, a new facility can only be established with its total capacity in one single period, while an existing facility can only be closed by reducing its total capacity at once. Hence, capacity is now "transferred" in modules with size equal to the full capacity of a given facility.

To install a new facility $j \in \mathcal{S}^o$ in some period t, it is required to "move" the desired capacity $\overline{K}_j^t$ from a fictitious facility i_0, similar to the capacity expansion scenario discussed in Section 4.1. This fictitious facility i_0 concentrates all the possible sizes of new facilities, that is, $\overline{K}_{i_0}^t = \sum_{j \in \mathcal{S}^o} \max_{\tau \in \mathcal{T}} \left\{ \overline{K}_j^\tau \right\}$ for $t \in \mathcal{T}$. The shutdown of an existing facility is modeled by "moving" its total capacity to a fictitious facility j_0 in a similar way as described in Section 4.2 for the capacity reduction case. Naturally, the capacity of an existing facility can no longer vary over time and thus, $\overline{K}_i^t = \overline{K}_i$ for every $i \in \mathcal{S}^c$ and $t \in \mathcal{T}$. Since the fictitious facility j_0 contains the total capacity of the facilities that are closed during the planning horizon, it follows that $\overline{K}_{j_0}^t = \sum_{i \in \mathcal{S}^c} \overline{K}_i$ for $t \in \mathcal{T}$. We note that i_0 and j_0 are not involved in any supply chain activities.

Using the location variables (23) and (24) of the alternative model (AP), the continuous variables $z_{i,j}^t$ for relocating capacity are now transformed into modular shifts:

$$z_{i,j_0}^t = \overline{K}_i^1 \; \eta_i^{t-1}, \qquad i \in \mathcal{S}^c,\, t \in \mathcal{T} \setminus \{1\} \tag{34}$$

$$z_{i_0,j}^t = \overline{K}_j^t \; \eta_j^t, \qquad j \in \mathcal{S}^o,\, t \in \mathcal{T}\,. \tag{35}$$

It is easy to see that constraints (5) and (7) are now redundant. Constraints (6) do not apply here since gradual capacity shifts to new facilities are no longer allowed. The modification of the capacity, minimum throughput and budget constraints (8), (9), (11), (14)–(16) is straightforward. Observe that since capacity transfers are restricted to opening and closing decisions, the moving costs $MC_{i_0,j}^t$ and MC_{i,j_0}^t are set to zero. Finally, constraints (4) and (10) remain unchanged, as well as the facility configuration conditions (27) (recall that the latter replace (12) and (13) of formulation (P)).

Classical facility location models consider in their objective functions fixed charges for installing and removing facilities, and activity-dependent costs (e.g. transportation costs). Although the former costs are taken into account in our budget constraints, it is easy to incorporate them in the objective function by redefining the operating costs OC_ℓ^t of each selectable facilities $\ell \in \mathcal{S}$. For each existing facility $i \in \mathcal{S}^c$ let $\overline{OC}_i^t = SC_i^{t+1}$ for every $t \in \setminus\{n\}$. Similarly, for a new facility $j \in \mathcal{S}^o$ we have $\overline{OC}_j^t = FC_j^{t-1}$ with $t \in \setminus\{1\}$. If desired, operating costs can also be incorporated in the new fixed costs by adding the terms $\sum_{\tau=1}^{t} OC_i^\tau$ and $\sum_{\tau=t}^{n} OC_j^\tau$, respectively.

We conclude this section by observing that it is relatively easy to further reduce our model to a simple dynamic location problem of the type analyzed, for example, by Van Roy and Erlenkotter [40]. A single echelon of uncapacitated facilities S, comprising both locations at which facilities may be opened and closed, is considered. Customers, corresponding to our non-selectable facilities, have demands for a single commodity. Since supply chain operations are restricted to transportation activities from facilities to customers, the flow conservation relations (4) reduce to demand satisfaction constraints. Furthermore, as it is usual in location models, fixed opening and closing costs are charged in the periods in which a facility starts operating or is shutdown, respectively. This entails allowing the facilities to be established or to be closed in the first and/or last periods of the planning horizon. Therefore, the time points marking the shutdown of an existing facility must be set at the beginning of each period t, in contrast to the definition given in (23). As result, the superscripts in the above definition of the new "operating costs" must be adjusted accordingly. Finally, our problem (P) is NP-hard since for $|\ \mathcal{T}\ |= 1$ and using the transformations described above it reduces to a static uncapacitated facility location problem which was shown to be NP-hard by Jacobsen [24].

Appendix B: Problem generation

For our computational experiments, test problems were randomly generated according to the three network configurations shown in Figure 3. To distinguish among the various types of facilities considered in the random tests, we introduce the following notation.

$\mathcal{C}$	:	set of customers
$\mathcal{F}$	:	set of plants
$\mathcal{S}$	:	set of (selectable) DCs
$\mathcal{S}_c^c$	:	set of existing central DCs
$\mathcal{S}_r^c$	:	set of existing regional DCs
$\mathcal{S}_c^o$	:	set of *new* central DCs
$\mathcal{S}_r^o$	:	set of *new* regional DCs

The set of all facilities is given by $\mathcal{L} = \mathcal{C} \cup \mathcal{F} \cup \mathcal{S}$. Furthermore, since relocation decisions are restricted to the distribution centers, it follows that all facilities that can be closed are represented by $\mathcal{S}^c = \mathcal{S}_c^c \cup \mathcal{S}_r^c$, while the potential new facilities are denoted by $\mathcal{S}^o = \mathcal{S}_c^o \cup \mathcal{S}_r^o$.

For each problem class, Table 4 indicates the number of elements in the above sets. It can be seen that the generated problems cover small and medium-scale networks. Unfortunately, larger problems could not be solved due to memory limitations. The second column of Table 4 gives the number of problems considered in each class. Since for each problem five instances were randomly generated, in total we performed our experiments on a set of 95 instances.

Table 5 details for each test problem the combinations selected with respect to the number of different facilities.

Table 4: Characteristics of the problem classes.

Problem class	# problems	Periods $\mid \mathcal{T} \mid$	Products $\mid \mathcal{P} \mid$	Non-selectable fac. $(\mathcal{L} \setminus \mathcal{S})$: Plants $\mid \mathcal{F} \mid$	Non-selectable fac. $(\mathcal{L} \setminus \mathcal{S})$: Customers $\mid \mathcal{C} \mid$	Selectable fac. $(\mathcal{S})$ Existing $(\mathcal{S}^c)$: Central DCs $\mid \mathcal{S}^c_c \mid$	Existing $(\mathcal{S}^c)$: Regional DCs $\mid \mathcal{S}^c_r \mid$	New $(\mathcal{S}^o)$: Central DCs $\mid \mathcal{S}^o_c \mid$	New $(\mathcal{S}^o)$: Regional DCs $\mid \mathcal{S}^o_r \mid$
1	9	3,4,5,6	5,10,15	-	50,75	10	-	20	-
2	6	4	5,10	5	50,75,100,150	10	-	20	-
3	4	3,4,5	5	5	50,75	8	12	4	8

Table 6 summarizes the values chosen for the generation of the parameters for problem classes 2 and 3, i.e. the demand requirements, cost factors, maximal capacities, etc. The generation of the instances in class 1 follows the same pattern as in class 2 except that in the former class the plant level is not present and so $\mathcal{F} = \varnothing$.

We denote by $U[a, b]$ the random generation of numbers in the interval $[a, b]$ according to a uniform distribution. Entries in Table 6 of the type $U[a, b](1 + U^{t-1}[c, d]\%)$ indicate that values between a and b were drawn from a uniform distribution in period $t = 1$. In each subsequent period $t \in \mathcal{T} \setminus \{1\}$, the generated value is not smaller than that of the previous period $t - 1$, varying the percentage increase between $c\%$ and $d\%$. For example, having $D^t_{\ell,p} = U[0, 25](1 + U^{t-1}[5, 10]\%)$ means that the demand of customer ℓ for product p in the first period is $D^1_{\ell,p} = U[0, 25]$. The demand for the second period is given by $D^2_{\ell,p} = D^1_{\ell,p} \times (1 + \gamma/100)$ with $\gamma \in U[0, 5]$. This means that a maximal increase of 5% can occur compared to the first period.

The intervals for the random generation of the parameters were selected in such a way that a large variety of instances were created that differ by the number of periods, products and facilities, the availability of transportation arcs, the range of fixed and variable costs, and the range of capacities and capital for investments. Finally, in an attempt to generate problems related to realistic cases, we restricted the amount of arcs available for the transportation of goods in the networks (see the bottom of Table 6). Also, only a given number of product types can actually flow through each generated arc. In this way, we can restrict the volume of traffic in the networks.

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Table 5: Combinations tested.

				Non-selectable fac. ($\mathcal{L} \setminus \mathcal{S}$)		Selectable fac. ($\mathcal{S}$) Existing ($\mathcal{S}^c$)		New ($\mathcal{S}^o$)	
Problem class	Problem id	Periods $\mid \mathcal{T} \mid$	Products $\mid \mathcal{P} \mid$	Plants $\mid \mathcal{F} \mid$	Customers $\mid \mathcal{C} \mid$	Central DCs $\mid \mathcal{S}^c_c \mid$	Regional DCs $\mid \mathcal{S}^c_r \mid$	Central DCs $\mid \mathcal{S}^o_c \mid$	Regional DCs $\mid \mathcal{S}^o_r \mid$
1	P1	3	5	-	50	10	-	20	-
	P2	4	5	-	50	10	-	20	-
	P3	5	5	-	50	10	-	20	-
	P4	6	5	-	50	10	-	20	-
	P5	3	10	-	50	10	-	20	-
	P6	3	15	-	50	10	-	20	-
	P7	4	5	-	75	10	-	20	-
	P8	4	10	-	50	10	-	20	-
	P9	4	10	-	75	10	-	20	-
2	P10	4	5	5	50	10	-	20	-
	P11	4	5	5	75	10	-	20	-
	P12	4	5	5	100	10	-	20	-
	P13	4	5	5	150	10	-	20	-
	P14	4	10	5	50	10	-	20	-
	P15	4	10	5	75	10	-	20	-
3	P16	3	5	5	50	8	12	4	8
	P17	4	5	5	50	8	12	4	8
	P18	5	5	5	50	8	12	4	8
	P19	3	5	5	75	8	12	4	8

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Table 6: Parameters selected for the random generation of the test problems.

Parameter	Symbol	Value: Problem class 2	Value: Problem class 3
Customer demand	$D^t_{\ell,p}$, $\ell \in \mathcal{C}$	$U[0,25](1+U^{t-1}[5,10]\%)$	$U[0,50](1+U^{t-1}[0,5]\%)$
Initial stock	$H_{\ell,p}$, $\ell \in \mathcal{F} \cup \mathcal{S}^c$	0	$U[0,8]$
Interest rate	α^t	$U[5,10]$	$U[5,7]$
Available budget	B^t	$U[20000,30000]$	$U[20000,30000]$
Costs:			
Purchasing/production	$PC^t_{\ell,p}$, $\ell \in \mathcal{F} \cup \mathcal{S}$	$U[30,150](1+U^{t-1}[5,10]\%)$	$U[15,25](1+U^{t-1}[0,5]\%)$
Transportation	$TC^t_{\ell,\ell',p}$, $\ell,\ell' \in \mathcal{L}$	$U[5,30](1+U^{t-1}[5,10]\%)$	$U[5,10](1+U^{t-1}[0,5]\%)$
Inventory carrying	$IC^t_{\ell,p}$, $\ell \in \mathcal{F} \cup \mathcal{S}$	$U[5,15](1+U^{t-1}[5,10]\%)$	$U[5,15](1+U^{t-1}[0,5]\%)$
Operating	OC^t_{ℓ}, $\ell \in \mathcal{F} \cup \mathcal{S}$	$U[700,1000](1+U^{t-1}[5,10]\%)$	$U[500,700](1-U^{t-1}[0,5]\%)$
Capacity relocation	$MC^t_{i,j}$, $i \in \mathcal{S}^c$, $j \in \mathcal{S}^o$	$U[50,75](1+U^{t-1}[5,10]\%)$	$U[25,50](1-U^{t-1}[0,5]\%)$
Closing (existing fac.)	SC^t_i, $i \in \mathcal{S}^c$		
central DCs	SC^t_i, $i \in \mathcal{S}^c_c$	$U[12000,30000](1+U^{t-1}[5,10]\%)$	$U[2000,3000](1-U^{t-1}[0,5]\%)$
regional DCs	SC^t_i, $i \in \mathcal{S}^c_r$		$U[1500,2000](1-U^{t-1}[0,5]\%)$
Opening (new fc.)	FC^t_j, $j \in \mathcal{S}^o$		
central DCs	FC^t_j, $j \in \mathcal{S}^o_c$	$U[10000,18000](1+U^{t-1}[5,10]\%)$	$U[1000,2000](1-U^{t-1}[0,5]\%)$
regional DCs	FC^t_j, $j \in \mathcal{S}^o_r$		$U[1000,1500](1-U^{t-1}[0,5]\%)$
Max. capacities:			
plants	$\overline{K}^t_\ell$, $\ell \in \mathcal{F}$	U[500,750]	$U[4000,5000]$
existing central DCs	$\overline{K}^t_i$, $i \in \mathcal{S}^c_c$	$U[150,300](1-U^{t-1}[0,7]\%)$	$U[2000,3500](1-U^{t-1}[0,5]\%)$
existing regional DCs	$\overline{K}^t_i$, $i \in \mathcal{S}^c_r$		$U[2000,3500](1+U^{t-1}[0,5]\%)$
new central DCs	$\overline{K}^t_j$, $j \in \mathcal{S}^o_c$	$U[150,300](1+U^{t-1}[0,7]\%)$	$U[1500,2000](1-U^{t-1}[0,5]\%)$
new regional DCs	$\overline{K}^t_j$, $j \in \mathcal{S}^o_r$		$U[1500,2000](1+U^{t-1}[0,5]\%)$
Min. throughput:	$\underline{K}^t_\ell$, $\ell \in \mathcal{S}$		
central DCs	$\underline{K}^t_\ell$, $\ell \in \mathcal{S}^c_c \cup \mathcal{S}^o_c$	$U[0,20](1+U^{t-1}[0,5]\%)$	$U[150,200](1+U^{t-1}[0,5]\%)$
regional DCs	$\underline{K}^t_\ell$, $\ell \in \mathcal{S}^c_r \cup \mathcal{S}^o_r$		$U[70,100](1+U^{t-1}[0,5]\%)$
Capacity consumption factor	$\mu_{\ell,p}$, $\ell \in \mathcal{F} \cup \mathcal{S}$	1	$U[1,2.5]$
Transp. arcs between facilities		50%	$50-70\%$
Products assigned to arcs		80%	70%

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Published reports of the Fraunhofer ITWM

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1. D. Hietel, K. Steiner, J. Struckmeier

A Finite - Volume Particle Method for Compressible Flows

We derive a new class of particle methods for conservation laws, which are based on numerical flux functions to model the interactions between moving particles. The derivation is similar to that of classical Finite-Volume methods; except that the fixed grid structure in the Finite-Volume method is substituted by so-called mass packets of particles. We give some numerical results on a shock wave solution for Burgers equation as well as the well-known one-dimensional shock tube problem.
(19 pages, 1998)

2. M. Feldmann, S. Seibold

Damage Diagnosis of Rotors: Application of Hilbert Transform and Multi-Hypothesis Testing

In this paper, a combined approach to damage diagnosis of rotors is proposed. The intention is to employ signal-based as well as model-based procedures for an improved detection of size and location of the damage. In a first step, Hilbert transform signal processing techniques allow for a computation of the signal envelope and the instantaneous frequency, so that various types of non-linearities due to a damage may be identified and classified based on measured response data. In a second step, a multi-hypothesis bank of Kalman Filters is employed for the detection of the size and location of the damage based on the information of the type of damage provided by the results of the Hilbert transform.
Keywords: Hilbert transform, damage diagnosis, Kalman filtering, non-linear dynamics
(23 pages, 1998)

3. Y. Ben-Haim, S. Seibold

Robust Reliability of Diagnostic Multi-Hypothesis Algorithms: Application to Rotating Machinery

Damage diagnosis based on a bank of Kalman filters, each one conditioned on a specific hypothesized system condition, is a well recognized and powerful diagnostic tool. This multi-hypothesis approach can be applied to a wide range of damage conditions. In this paper, we will focus on the diagnosis of cracks in rotating machinery. The question we address is: how to optimize the multi-hypothesis algorithm with respect to the uncertainty of the spatial form and location of cracks and their resulting dynamic effects. First, we formulate a measure of the reliability of the diagnostic algorithm, and then we discuss modifications of the diagnostic algorithm for the maximization of the reliability. The reliability of a diagnostic algorithm is measured by the amount of uncertainty consistent with no-failure of the diagnosis. Uncertainty is quantitatively represented with convex models.
Keywords: Robust reliability, convex models, Kalman filtering, multi-hypothesis diagnosis, rotating machinery, crack diagnosis
(24 pages, 1998)

4. F.-Th. Lentes, N. Siedow

Three-dimensional Radiative Heat Transfer in Glass Cooling Processes

For the numerical simulation of 3D radiative heat transfer in glasses and glass melts, practically applicable mathematical methods are needed to handle such problems optimal using workstation class computers. Since the exact solution would require super-computer capabilities we concentrate on approximate solutions with a high degree of accuracy. The following approaches are studied: 3D diffusion approximations and 3D ray-tracing methods.
(23 pages, 1998)

5. A. Klar, R. Wegener

A hierarchy of models for multilane vehicular traffic
Part I: Modeling

In the present paper multilane models for vehicular traffic are considered. A microscopic multilane model based on reaction thresholds is developed. Based on this model an Enskog like kinetic model is developed. In particular, care is taken to incorporate the correlations between the vehicles. From the kinetic model a fluid dynamic model is derived. The macroscopic coefficients are deduced from the underlying kinetic model. Numerical simulations are presented for all three levels of description in [10]. Moreover, a comparison of the results is given there.
(23 pages, 1998)

Part II: Numerical and stochastic investigations

In this paper the work presented in [6] is continued. The present paper contains detailed numerical investigations of the models developed there. A numerical method to treat the kinetic equations obtained in [6] are presented and results of the simulations are shown. Moreover, the stochastic correlation model used in [6] is described and investigated in more detail.
(17 pages, 1998)

6. A. Klar, N. Siedow

Boundary Layers and Domain Decomposition for Radiative Heat Transfer and Diffusion Equations: Applications to Glass Manufacturing Processes

In this paper domain decomposition methods for radiative transfer problems including conductive heat transfer are treated. The paper focuses on semi-transparent materials, like glass, and the associated conditions at the interface between the materials. Using asymptotic analysis we derive conditions for the coupling of the radiative transfer equations and a diffusion approximation. Several test cases are treated and a problem appearing in glass manufacturing processes is computed. The results clearly show the advantages of a domain decomposition approach. Accuracy equivalent to the solution of the global radiative transfer solution is achieved, whereas computation time is strongly reduced.
(24 pages, 1998)

7. I. Choquet

Heterogeneous catalysis modelling and numerical simulation in rarified gas flows
Part I: Coverage locally at equilibrium

A new approach is proposed to model and simulate numerically heterogeneous catalysis in rarefied gas flows. It is developed to satisfy all together the following points:
1) describe the gas phase at the microscopic scale, as required in rarefied flows,
2) describe the wall at the macroscopic scale, to avoid prohibitive computational costs and consider not only crystalline but also amorphous surfaces,
3) reproduce on average macroscopic laws correlated with experimental results and
4) derive analytic models in a systematic and exact way. The problem is stated in the general framework of a non static flow in the vicinity of a catalytic and non porous surface (without aging). It is shown that the exact and systematic resolution method based on the Laplace transform, introduced previously by the author to model collisions in the gas phase, can be extended to the present problem. The proposed approach is applied to the modelling of the EleyRideal and LangmuirHinshelwood recombinations, assuming that the coverage is locally at equilibrium. The models are developed considering one atomic species and extended to the general case of several atomic species. Numerical calculations show that the models derived in this way reproduce with accuracy behaviors observed experimentally.
(24 pages, 1998)

8. J. Ohser, B. Steinbach, C. Lang

Efficient Texture Analysis of Binary Images

A new method of determining some characteristics of binary images is proposed based on a special linear filtering. This technique enables the estimation of the area fraction, the specific line length, and the specific integral of curvature. Furthermore, the specific length of the total projection is obtained, which gives detailed information about the texture of the image. The influence of lateral and directional resolution depending on the size of the applied filter mask is discussed in detail. The technique includes a method of increasing directional resolution for texture analysis while keeping lateral resolution as high as possible.
(17 pages, 1998)

9. J. Orlik

Homogenization for viscoelasticity of the integral type with aging and shrinkage

A multiphase composite with periodic distributed inclusions with a smooth boundary is considered in this contribution. The composite component materials are supposed to be linear viscoelastic and aging (of the nonconvolution integral type, for which the Laplace transform with respect to time is not effectively applicable) and are subjected to isotropic shrinkage. The free shrinkage deformation can be considered as a fictitious temperature deformation in the behavior law. The procedure presented in this paper proposes a way to determine average (effective homogenized) viscoelastic and shrinkage (temperature) composite properties and the homogenized stressfield from known properties of the components. This is done by the extension of the asymptotic homogenization technique known for pure elastic nonhomogeneous bodies to the nonhomogeneous thermoviscoelasticity of the integral noncon-

volution type. Up to now, the homogenization theory has not covered viscoelasticity of the integral type. SanchezPalencia (1980), Francfort & Suquet (1987) (see [2], [9]) have considered homogenization for viscoelasticity of the differential form and only up to the first derivative order. The integralmodeled viscoelasticity is more general then the differential one and includes almost all known differential models. The homogenization procedure is based on the construction of an asymptotic solution with respect to a period of the composite structure. This reduces the original problem to some auxiliary boundary value problems of elasticity and viscoelasticity on the unit periodic cell, of the same type as the original non-homogeneous problem. The existence and uniqueness results for such problems were obtained for kernels satisfying some constrain conditions. This is done by the extension of the Volterra integral operator theory to the Volterra operators with respect to the time, whose 1 kernels are space linear operators for any fixed time variables. Some ideas of such approach were proposed in [11] and [12], where the Volterra operators with kernels depending additionally on parameter were considered. This manuscript delivers results of the same nature for the case of the spaceoperator kernels.
(20 pages, 1998)

10. J. Mohring

Helmholtz Resonators with Large Aperture

The lowest resonant frequency of a cavity resonator is usually approximated by the classical Helmholtz formula. However, if the opening is rather large and the front wall is narrow this formula is no longer valid. Here we present a correction which is of third order in the ratio of the diameters of aperture and cavity. In addition to the high accuracy it allows to estimate the damping due to radiation. The result is found by applying the method of matched asymptotic expansions. The correction contains form factors describing the shapes of opening and cavity. They are computed for a number of standard geometries. Results are compared with numerical computations.
(21 pages, 1998)

11. H. W. Hamacher, A. Schöbel

On Center Cycles in Grid Graphs

Finding "good" cycles in graphs is a problem of great interest in graph theory as well as in locational analysis. We show that the center and median problems are NP hard in general graphs. This result holds both for the variable cardinality case (i.e. all cycles of the graph are considered) and the fixed cardinality case (i.e. only cycles with a given cardinality p are feasible). Hence it is of interest to investigate special cases where the problem is solvable in polynomial time. In grid graphs, the variable cardinality case is, for instance, trivially solvable if the shape of the cycle can be chosen freely. If the shape is fixed to be a rectangle one can analyze rectangles in grid graphs with, in sequence, fixed dimension, fixed cardinality, and variable cardinality. In all cases a complete characterization of the optimal cycles and closed form expressions of the optimal objective values are given, yielding polynomial time algorithms for all cases of center rectangle problems. Finally, it is shown that center cycles can be chosen as rectangles for small cardinalities such that the center cycle problem in grid graphs is in these cases completely solved.
(15 pages, 1998)

12. H. W. Hamacher, K.-H. Küfer

Inverse radiation therapy planning - a multiple objective optimisation approach

For some decades radiation therapy has been proved successful in cancer treatment. It is the major task of clinical radiation treatment planning to realize on the one hand a high level dose of radiation in the cancer tissue in order to obtain maximum tumor control. On the other hand it is obvious that it is absolutely necessary to keep in the tissue outside the tumor, particularly in organs at risk, the unavoidable radiation as low as possible.
No doubt, these two objectives of treatment planning - high level dose in the tumor, low radiation outside the tumor - have a basically contradictory nature. Therefore, it is no surprise that inverse mathematical models with dose distribution bounds tend to be infeasible in most cases. Thus, there is need for approximations compromising between overdosing the organs at risk and underdosing the target volume.
Differing from the currently used time consuming iterative approach, which measures deviation from an ideal (non-achievable) treatment plan using recursively trial-and-error weights for the organs of interest, we go a new way trying to avoid a priori weight choices and consider the treatment planning problem as a multiple objective linear programming problem: with each organ of interest, target tissue as well as organs at risk, we associate an objective function measuring the maximal deviation from the prescribed doses.
We build up a data base of relatively few efficient solutions representing and approximating the variety of Pareto solutions of the multiple objective linear programming problem. This data base can be easily scanned by physicians looking for an adequate treatment plan with the aid of an appropriate online tool.
(14 pages, 1999)

13. C. Lang, J. Ohser, R. Hilfer

On the Analysis of Spatial Binary Images

This paper deals with the characterization of microscopically heterogeneous, but macroscopically homogeneous spatial structures. A new method is presented which is strictly based on integral-geometric formulae such as Crofton's intersection formulae and Hadwiger's recursive definition of the Euler number. The corresponding algorithms have clear advantages over other techniques. As an example of application we consider the analysis of spatial digital images produced by means of Computer Assisted Tomography.
(20 pages, 1999)

14. M. Junk

On the Construction of Discrete Equilibrium Distributions for Kinetic Schemes

A general approach to the construction of discrete equilibrium distributions is presented. Such distribution functions can be used to set up Kinetic Schemes as well as Lattice Boltzmann methods. The general principles are also applied to the construction of Chapman Enskog distributions which are used in Kinetic Schemes for compressible Navier-Stokes equations.
(24 pages, 1999)

15. M. Junk, S. V. Raghurame Rao

A new discrete velocity method for Navier-Stokes equations

The relation between the Lattice Boltzmann Method, which has recently become popular, and the Kinetic Schemes, which are routinely used in Computational Fluid Dynamics, is explored. A new discrete velocity model for the numerical solution of Navier-Stokes equations for incompressible fluid flow is presented by combining both the approaches. The new scheme can be interpreted as a pseudo-compressibility method and, for a particular choice of parameters, this interpretation carries over to the Lattice Boltzmann Method.
(20 pages, 1999)

16. H. Neunzert

Mathematics as a Key to Key Technologies

The main part of this paper will consist of examples, how mathematics really helps to solve industrial problems; these examples are taken from our Institute for Industrial Mathematics, from research in the Technomathematics group at my university, but also from ECMI groups and a company called TecMath, which originated 10 years ago from my university group and has already a very successful history.
(39 pages (4 PDF-Files), 1999)

17. J. Ohser, K. Sandau

Considerations about the Estimation of the Size Distribution in Wicksell's Corpuscle Problem

Wicksell's corpuscle problem deals with the estimation of the size distribution of a population of particles, all having the same shape, using a lower dimensional sampling probe. This problem was originary formulated for particle systems occurring in life sciences but its solution is of actual and increasing interest in materials science. From a mathematical point of view, Wicksell's problem is an inverse problem where the interesting size distribution is the unknown part of a Volterra equation. The problem is often regarded ill-posed, because the structure of the integrand implies unstable numerical solutions. The accuracy of the numerical solutions is considered here using the condition number, which allows to compare different numerical methods with different (equidistant) class sizes and which indicates, as one result, that a finite section thickness of the probe reduces the numerical problems. Furthermore, the relative error of estimation is computed which can be split into two parts. One part consists of the relative discretization error that increases for increasing class size, and the second part is related to the relative statistical error which increases with decreasing class size. For both parts, upper bounds can be given and the sum of them indicates an optimal class width depending on some specific constants.
(18 pages, 1999)

18. E. Carrizosa, H. W. Hamacher, R. Klein, S. Nickel

Solving nonconvex planar location problems by finite dominating sets

It is well-known that some of the classical location problems with polyhedral gauges can be solved in polynomial time by finding a finite dominating set, i. e. a finite set of candidates guaranteed to contain at least one optimal location.
In this paper it is first established that this result holds

for a much larger class of problems than currently considered in the literature. The model for which this result can be proven includes, for instance, location problems with attraction and repulsion, and location-allocation problems.
Next, it is shown that the approximation of general gauges by polyhedral ones in the objective function of our general model can be analyzed with regard to the subsequent error in the optimal objective value. For the approximation problem two different approaches are described, the sandwich procedure and the greedy algorithm. Both of these approaches lead - for fixed epsilon - to polynomial approximation algorithms with accuracy epsilon for solving the general model considered in this paper.
Keywords: Continuous Location, Polyhedral Gauges, Finite Dominating Sets, Approximation, Sandwich Algorithm, Greedy Algorithm
(19 pages, 2000)

19. A. Becker

A Review on Image Distortion Measures

Within this paper we review image distortion measures. A distortion measure is a criterion that assigns a "quality number" to an image. We distinguish between mathematical distortion measures and those distortion measures in-cooperating a priori knowledge about the imaging devices (e.g. satellite images), image processing algorithms or the human physiology. We will consider representative examples of different kinds of distortion measures and are going to discuss them.
Keywords: Distortion measure, human visual system
(26 pages, 2000)

20. H. W. Hamacher, M. Labbé, S. Nickel, T. Sonneborn

Polyhedral Properties of the Uncapacitat*ed Multiple Allocation Hub Location Problem*

We examine the feasibility polyhedron of the uncapacitated hub location problem (UHL) with multiple allocation, which has applications in the fields of air passenger and cargo transportation, telecommunication and postal delivery services. In particular we determine the dimension and derive some classes of facets of this polyhedron. We develop some general rules about lifting facets from the uncapacitated facility location (UFL) for UHL and projecting facets from UHL to UFL. By applying these rules we get a new class of facets for UHL which dominates the inequalities in the original formulation. Thus we get a new formulation of UHL whose constraints are all facet–defining. We show its superior computational performance by benchmarking it on a well known data set.
Keywords: integer programming, hub location, facility location, valid inequalities, facets, branch and cut
(21 pages, 2000)

21. H. W. Hamacher, A. Schöbel

Design of Zone Tariff Systems in Public Transportation

Given a public transportation system represented by its stops and direct connections between stops, we consider two problems dealing with the prices for the customers: The fare problem in which subsets of stops are already aggregated to zones and "good" tariffs have to be found in the existing zone system. Closed form solutions for the fare problem are presented for three objective functions. In the zone problem the design of the zones is part of the problem. This problem is NP hard and we therefore propose three heuristics which prove to be very successful in the redesign of one of Germany's transportation systems.
(30 pages, 2001)

22. D. Hietel, M. Junk, R. Keck, D. Teleaga:

The Finite-Volume-Particle Method for Conservation Laws

In the Finite-Volume-Particle Method (FVPM), the weak formulation of a hyperbolic conservation law is discretized by restricting it to a discrete set of test functions. In contrast to the usual Finite-Volume approach, the test functions are not taken as characteristic functions of the control volumes in a spatial grid, but are chosen from a partition of unity with smooth and overlapping partition functions (the particles), which can even move along pre- scribed velocity fields. The information exchange between particles is based on standard numerical flux functions. Geometrical information, similar to the surface area of the cell faces in the Finite-Volume Method and the corresponding normal directions are given as integral quantities of the partition functions. After a brief derivation of the Finite-Volume-Particle Method, this work focuses on the role of the geometric coefficients in the scheme.
(16 pages, 2001)

23. T. Bender, H. Hennes, J. Kalcsics, M. T. Melo, S. Nickel

Location Software and Interface with GIS and Supply Chain Management

The objective of this paper is to bridge the gap between location theory and practice. To meet this objective focus is given to the development of software capable of addressing the different needs of a wide group of users. There is a very active community on location theory encompassing many research fields such as operations research, computer science, mathematics, engineering, geography, economics and marketing. As a result, people working on facility location problems have a very diverse background and also different needs regarding the software to solve these problems. For those interested in non-commercial applications (e. g. students and researchers), the library of location algorithms (LoLA can be of considerable assistance. LoLA contains a collection of efficient algorithms for solving planar, network and discrete facility location problems. In this paper, a detailed description of the functionality of LoLA is presented. In the fields of geography and marketing, for instance, solving facility location problems requires using large amounts of demographic data. Hence, members of these groups (e. g. urban planners and sales managers) often work with geographical information too s. To address the specific needs of these users, LoLA was inked to a geographical information system (GIS) and the details of the combined functionality are described in the paper. Finally, there is a wide group of practitioners who need to solve large problems and require special purpose software with a good data interface. Many of such users can be found, for example, in the area of supply chain management (SCM). Logistics activities involved in strategic SCM include, among others, facility location planning. In this paper, the development of a commercial location software tool is also described. The too is embedded in the Advanced Planner and Optimizer SCM software developed by SAP AG, Walldorf, Germany. The paper ends with some conclusions and an outlook to future activities.
Keywords: facility location, software development, geographical information systems, supply chain management.
(48 pages, 2001)

24. H. W. Hamacher, S. A. Tjandra

Mathematical Modelling of Evacuation Problems: A State of Art

This paper details models and algorithms which can be applied to evacuation problems. While it concentrates on building evacuation many of the results are applicable also to regional evacuation. All models consider the time as main parameter, where the travel time between components of the building is part of the input and the overall evacuation time is the output. The paper distinguishes between macroscopic and microscopic evacuation models both of which are able to capture the evacuees' movement over time.
Macroscopic models are mainly used to produce good lower bounds for the evacuation time and do not consider any individual behavior during the emergency situation. These bounds can be used to analyze existing buildings or help in the design phase of planning a building. Macroscopic approaches which are based on dynamic network flow models (minimum cost dynamic flow, maximum dynamic flow, universal maximum flow, quickest path and quickest flow) are described. A special feature of the presented approach is the fact, that travel times of evacuees are not restricted to be constant, but may be density dependent. Using multicriteria optimization priority regions and blockage due to fire or smoke may be considered. It is shown how the modelling can be done using time parameter either as discrete or continuous parameter.
Microscopic models are able to model the individual evacuee's characteristics and the interaction among evacuees which influence their movement. Due to the corresponding huge amount of data one uses simulation approaches. Some probabilistic laws for individual evacuee's movement are presented. Moreover ideas to model the evacuee's movement using cellular automata (CA) and resulting software are presented.
In this paper we will focus on macroscopic models and only summarize some of the results of the microscopic approach. While most of the results are applicable to general evacuation situations, we concentrate on building evacuation.
(44 pages, 2001)

25. J. Kuhnert, S. Tiwari

Grid free method for solving the Poisson equation

A Grid free method for solving the Poisson equation is presented. This is an iterative method. The method is based on the weighted least squares approximation in which the Poisson equation is enforced to be satisfied in every iterations. The boundary conditions can also be enforced in the iteration process. This is a local approximation procedure. The Dirichlet, Neumann and mixed boundary value problems on a unit square are presented and the analytical solutions are compared with the exact solutions. Both solutions matched perfectly.
Keywords: Poisson equation, Least squares method, Grid free method
(19 pages, 2001)

26. T. Götz, H. Rave, D. Reinel-Bitzer, K. Steiner, H. Tiemeier

Simulation of the fiber spinning process

To simulate the influence of process parameters to the melt spinning process a fiber model is used and coupled with CFD calculations of the quench air flow. In the fiber model energy, momentum and mass balance are solved for the polymer mass flow. To calculate the quench air the Lattice Boltzmann method is used. Simulations and experiments for different process parameters and hole configurations are compared and show a good agreement.
Keywords: Melt spinning, fiber model, Lattice Boltzmann, CFD
(19 pages, 2001)

27. A. Zemitis

On interaction of a liquid film with an obstacle

In this paper mathematical models for liquid films generated by impinging jets are discussed. Attention is stressed to the interaction of the liquid film with some obstacle. S. G. Taylor [Proc. R. Soc. London Ser. A 253, 313 (1959)] found that the liquid film generated by impinging jets is very sensitive to properties of the wire which was used as an obstacle. The aim of this presentation is to propose a modification of the Taylor's model, which allows to simulate the film shape in cases, when the angle between jets is different from 180°. Numerical results obtained by discussed models give two different shapes of the liquid film similar as in Taylors experiments. These two shapes depend on the regime: either droplets are produced close to the obstacle or not. The difference between two regimes becomes larger if the angle between jets decreases. Existence of such two regimes can be very essential for some applications of impinging jets, if the generated liquid film can have a contact with obstacles.
Keywords: impinging jets, liquid film, models, numerical solution, shape
(22 pages, 2001)

28. I. Ginzburg, K. Steiner

Free surface lattice-Boltzmann method to model the filling of expanding cavities by Bingham Fluids

The filling process of viscoplastic metal alloys and plastics in expanding cavities is modelled using the lattice Boltzmann method in two and three dimensions. These models combine the regularized Bingham model for viscoplastic with a free-interface algorithm. The latter is based on a modified immiscible lattice Boltzmann model in which one species is the fluid and the other one is considered as vacuum. The boundary conditions at the curved liquid-vacuum interface are met without any geometrical front reconstruction from a first-order Chapman-Enskog expansion. The numerical results obtained with these models are found in good agreement with available theoretical and numerical analysis.
Keywords: Generalized LBE, free-surface phenomena, interface boundary conditions, filling processes, Bingham viscoplastic model, regularized models
(22 pages, 2001)

29. H. Neunzert

»Denn nichts ist für den Menschen als Menschen etwas wert, was er nicht mit Leidenschaft tun kann«

Vortrag anlässlich der Verleihung des Akademiepreises des Landes Rheinland-Pfalz am 21.11.2001

Was macht einen guten Hochschullehrer aus? Auf diese Frage gibt es sicher viele verschiedene, fachbezogene Antworten, aber auch ein paar allgemeine Gesichtspunkte: es bedarf der »Leidenschaft« für die Forschung (Max Weber), aus der dann auch die Begeisterung für die Lehre erwächst. Forschung und Lehre gehören zusammen, um die Wissenschaft als lebendiges Tun vermitteln zu können. Der Vortrag gibt Beispiele dafür, wie in angewandter Mathematik Forschungsaufgaben aus praktischen Alltagsproblemstellungen erwachsen, die in die Lehre auf verschiedenen Stufen (Gymnasium bis Graduiertenkolleg) einfließen; er leitet damit auch zu einem aktuellen Forschungsgebiet, der Mehrskalenanalyse mit ihren vielfältigen Anwendungen in Bildverarbeitung, Materialentwicklung und Strömungsmechanik über, was aber nur kurz gestreift wird. Mathematik erscheint hier als eine moderne Schlüsseltechnologie, die aber auch enge Beziehungen zu den Geistes- und Sozialwissenschaften hat.
Keywords: Lehre, Forschung, angewandte Mathematik, Mehrskalenanalyse, Strömungsmechanik
(18 pages, 2001)

30. J. Kuhnert, S. Tiwari

Finite pointset method based on the projection method for simulations of the incompressible Navier-Stokes equations

A Lagrangian particle scheme is applied to the projection method for the incompressible Navier-Stokes equations. The approximation of spatial derivatives is obtained by the weighted least squares method. The pressure Poisson equation is solved by a local iterative procedure with the help of the least squares method. Numerical tests are performed for two dimensional cases. The Couette flow, Poiseuelle flow, decaying shear flow and the driven cavity flow are presented. The numerical solutions are obtained for stationary as well as instationary cases and are compared with the analytical solutions for channel flows. Finally, the driven cavity in a unit square is considered and the stationary solution obtained from this scheme is compared with that from the finite element method.
Keywords: Incompressible Navier-Stokes equations, Meshfree method, Projection method, Particle scheme, Least squares approximation
AMS subject classification: 76D05, 76M28
(25 pages, 2001)

31. R. Korn, M. Krekel

Optimal Portfolios with Fixed Consumption or Income Streams

We consider some portfolio optimisation problems where either the investor has a desire for an a priori specified consumption stream or/and follows a deterministic pay in scheme while also trying to maximize expected utility from final wealth. We derive explicit closed form solutions for continuous and discrete monetary streams. The mathematical method used is classical stochastic control theory.
Keywords: Portfolio optimisation, stochastic control, HJB equation, discretisation of control problems.
(23 pages, 2002)

32. M. Krekel

Optimal portfolios with a loan dependent credit spread

If an investor borrows money he generally has to pay higher interest rates than he would have received, if he had put his funds on a savings account. The classical model of continuous time portfolio optimisation ignores this effect. Since there is obviously a connection between the default probability and the total percentage of wealth, which the investor is in debt, we study portfolio optimisation with a control dependent interest rate. Assuming a logarithmic and a power utility function, respectively, we prove explicit formulae of the optimal control.
Keywords: Portfolio optimisation, stochastic control, HJB equation, credit spread, log utility, power utility, non-linear wealth dynamics
(25 pages, 2002)

33. J. Ohser, W. Nagel, K. Schladitz

The Euler number of discretized sets - on the choice of adjacency in homogeneous lattices

Two approaches for determining the Euler-Poincaré characteristic of a set observed on lattice points are considered in the context of image analysis { the integral geometric and the polyhedral approach. Information about the set is assumed to be available on lattice points only. In order to retain properties of the Euler number and to provide a good approximation of the true Euler number of the original set in the Euclidean space, the appropriate choice of adjacency in the lattice for the set and its background is crucial. Adjacencies are defined using tessellations of the whole space into polyhedrons. In R 3 , two new 14 adjacencies are introduced additionally to the well known 6 and 26 adjacencies. For the Euler number of a set and its complement, a consistency relation holds. Each of the pairs of adjacencies (14:1; 14:1), (14:2; 14:2), (6; 26), and (26; 6) is shown to be a pair of complementary adjacencies with respect to this relation. That is, the approximations of the Euler numbers are consistent if the set and its background (complement) are equipped with this pair of adjacencies. Furthermore, sufficient conditions for the correctness of the approximations of the Euler number are given. The analysis of selected microstructures and a simulation study illustrate how the estimated Euler number depends on the chosen adjacency. It also shows that there is not a uniquely best pair of adjacencies with respect to the estimation of the Euler number of a set in Euclidean space.
Keywords: image analysis, Euler number, neighborhod relationships, cuboidal lattice
(32 pages, 2002)

34. I. Ginzburg, K. Steiner

Lattice Boltzmann Model for Free-Surface flow and Its Application to Filling Process in Casting

A generalized lattice Boltzmann model to simulate free-surface is constructed in both two and three dimensions. The proposed model satisfies the interfacial boundary conditions accurately. A distinctive feature of the model is that the collision processes is carried out only on the points occupied partially or fully by the fluid. To maintain a sharp interfacial front, the method includes an anti-diffusion algorithm. The unknown distribution functions at the interfacial region are constructed according to the first order Chapman-Enskog analysis. The interfacial boundary conditions are satis-

fied exactly by the coefficients in the Chapman-Enskog expansion. The distribution functions are naturally expressed in the local interfacial coordinates. The macroscopic quantities at the interface are extracted from the least-square solutions of a locally linearized system obtained from the known distribution functions. The proposed method does not require any geometric front construction and is robust for any interfacial topology. Simulation results of realistic filling process are presented: rectangular cavity in two dimensions and Hammer box, Campbell box, Sheffield box, and Motorblock in three dimensions. To enhance the stability at high Reynolds numbers, various upwind-type schemes are developed. Free-slip and no-slip boundary conditions are also discussed.
Keywords: Lattice Boltzmann models; free-surface phenomena; interface boundary conditions; filling processes; injection molding; volume of fluid method; interface boundary conditions; advection-schemes; upwind-schemes
(54 pages, 2002)

35. M. Günther, A. Klar, T. Materne, R. Wegener

Multivalued fundamental diagrams and stop and go waves for continuum traffic equations

In the present paper a kinetic model for vehicular traffic leading to multivalued fundamental diagrams is developed and investigated in detail. For this model phase transitions can appear depending on the local density and velocity of the flow. A derivation of associated macroscopic traffic equations from the kinetic equation is given. Moreover, numerical experiments show the appearance of stop and go waves for highway traffic with a bottleneck.
Keywords: traffic flow, macroscopic equations, kinetic derivation, multivalued fundamental diagram, stop and go waves, phase transitions
(25 pages, 2002)

36. S. Feldmann, P. Lang, D. Prätzel-Wolters

Parameter influence on the zeros of network determinants

To a network N(q) with determinant D(s;q) depending on a parameter vector q Î R^r via identification of some of its vertices, a network N^ (q) is assigned. The paper deals with procedures to find N^ (q), such that its determinant D^ (s;q) admits a factorization in the determinants of appropriate subnetworks, and with the estimation of the deviation of the zeros of D^ from the zeros of D. To solve the estimation problem state space methods are applied.
Keywords: Networks, Equicofactor matrix polynomials, Realization theory, Matrix perturbation theory
(30 pages, 2002)

37. K. Koch, J. Ohser, K. Schladitz

Spectral theory for random closed sets and estimating the covariance via frequency space

A spectral theory for stationary random closed sets is developed and provided with a sound mathematical basis. Definition and proof of existence of the Bartlett spectrum of a stationary random closed set as well as the proof of a Wiener-Khintchine theorem for the power spectrum are used to two ends: First, well known second order characteristics like the covariance can be estimated faster than usual via frequency space. Second, the Bartlett spectrum and the power spectrum can be used as second order characteristics in frequency space. Examples show, that in some cases information about the random closed set is easier to obtain from these characteristics in frequency space than from their real world counterparts.
Keywords: Random set, Bartlett spectrum, fast Fourier transform, power spectrum
(28 pages, 2002)

38. D. d'Humières, I. Ginzburg

Multi-reflection boundary conditions for lattice Boltzmann models

We present a unified approach of several boundary conditions for lattice Boltzmann models. Its general framework is a generalization of previously introduced schemes such as the bounce-back rule, linear or quadratic interpolations, etc. The objectives are two fold: first to give theoretical tools to study the existing boundary conditions and their corresponding accuracy; secondly to design formally third- order accurate boundary conditions for general flows. Using these boundary conditions, Couette and Poiseuille flows are exact solution of the lattice Boltzmann models for a Reynolds number $Re = 0$ (Stokes limit).
Numerical comparisons are given for Stokes flows in periodic arrays of spheres and cylinders, linear periodic array of cylinders between moving plates and for Navier-Stokes flows in periodic arrays of cylinders for $Re < 200$. These results show a significant improvement of the overall accuracy when using the linear interpolations instead of the bounce-back reflection (up to an order of magnitude on the hydrodynamics fields). Further improvement is achieved with the new multi-reflection boundary conditions, reaching a level of accuracy close to the quasi-analytical reference solutions, even for rather modest grid resolutions and few points in the narrowest channels. More important, the pressure and velocity fields in the vicinity of the obstacles are much smoother with multi-reflection than with the other boundary conditions.
Finally the good stability of these schemes is highlighted by some simulations of moving obstacles: a cylinder between flat walls and a sphere in a cylinder.
Keywords: lattice Boltzmann equation, boudary condistions, bounce-back rule, Navier-Stokes equation
(72 pages, 2002)

39. R. Korn

Elementare Finanzmathematik

Im Rahmen dieser Arbeit soll eine elementar gehaltene Einführung in die Aufgabenstellungen und Prinzipien der modernen Finanzmathematik gegeben werden. Insbesondere werden die Grundlagen der Modellierung von Aktienkursen, der Bewertung von Optionen und der Portfolio-Optimierung vorgestellt. Natürlich können die verwendeten Methoden und die entwickelte Theorie nicht in voller Allgemeinheit für den Schulunterricht verwendet werden, doch sollen einzelne Prinzipien so heraus gearbeitet werden, dass sie auch an einfachen Beispielen verstanden werden können.
Keywords: Finanzmathematik, Aktien, Optionen, Portfolio-Optimierung, Börse, Lehrerweiterbildung, Mathematikunterricht
(98 pages, 2002)

40. J. Kallrath, M. C. Müller, S. Nickel

Batch Presorting Problems: Models and Complexity Results

In this paper we consider short term storage systems. We analyze presorting strategies to improve the effiency of these storage systems. The presorting task is called Batch PreSorting Problem (BPSP). The BPSP is a variation of an assigment problem, i.e., it has an assigment problem kernel and some additional constraints. We present different types of these presorting problems, introduce mathematical programming formulations and prove the NP-completeness for one type of the BPSP. Experiments are carried out in order to compare the different model formulations and to investigate the behavior of these models.
Keywords: Complexity theory, Integer programming, Assigment, Logistics
(19 pages, 2002)

41. J. Linn

On the frame-invariant description of the phase space of the Folgar-Tucker equation

The Folgar-Tucker equation is used in flow simulations of fiber suspensions to predict fiber orientation depending on the local flow. In this paper, a complete, frame-invariant description of the phase space of this differential equation is presented for the first time.
Key words: fiber orientation, Folgar-Tucker equation, injection molding
(5 pages, 2003)

42. T. Hanne, S. Nickel

A Multi-Objective Evolutionary Algorithm for Scheduling and Inspection Planning in Software Development Projects

In this article, we consider the problem of planning inspections and other tasks within a software development (SD) project with respect to the objectives quality (no. of defects), project duration, and costs. Based on a discrete-event simulation model of SD processes comprising the phases coding, inspection, test, and rework, we present a simplified formulation of the problem as a multiobjective optimization problem. For solving the problem (i.e. finding an approximation of the efficient set) we develop a multiobjective evolutionary algorithm. Details of the algorithm are discussed as well as results of its application to sample problems.
Key words: multiple objective programming, project management and scheduling, software development, evolutionary algorithms, efficient set
(29 pages, 2003)

43. T. Bortfeld , K.-H. Küfer, M. Monz, A. Scherrer, C. Thieke, H. Trinkaus

Intensity-Modulated Radiotherapy - A Large Scale Multi-Criteria Programming Problem -

Radiation therapy planning is always a tight rope walk between dangerous insufficient dose in the target volume and life threatening overdosing of organs at risk. Finding ideal balances between these inherently contradictory goals challenges dosimetrists and physicians in their daily practice. Today's planning systems are typically based on a single evaluation function that measures the quality of a radiation treatment plan. Unfortunately, such a one dimensional approach can-

not satisfactorily map the different backgrounds of physicians and the patient dependent necessities. So, too often a time consuming iteration process between evaluation of dose distribution and redefinition of the evaluation function is needed.
In this paper we propose a generic multi-criteria approach based on Pareto's solution concept. For each entity of interest - target volume or organ at risk a structure dependent evaluation function is defined measuring deviations from ideal doses that are calculated from statistical functions. A reasonable bunch of clinically meaningful Pareto optimal solutions are stored in a data base, which can be interactively searched by physicians. The system guarantees dynamical planning as well as the discussion of tradeoffs between different entities.
Mathematically, we model the upcoming inverse problem as a multi-criteria linear programming problem. Because of the large scale nature of the problem it is not possible to solve the problem in a 3D-setting without adaptive reduction by appropriate approximation schemes.
Our approach is twofold: First, the discretization of the continuous problem is based on an adaptive hierarchical clustering process which is used for a local refinement of constraints during the optimization procedure. Second, the set of Pareto optimal solutions is approximated by an adaptive grid of representatives that are found by a hybrid process of calculating extreme compromises and interpolation methods.
Keywords: multiple criteria optimization, representative systems of Pareto solutions, adaptive triangulation, clustering and disaggregation techniques, visualization of Pareto solutions, medical physics, external beam radiotherapy planning, intensity modulated radiotherapy
(31 pages, 2003)

44. T. Halfmann, T. Wichmann

Overview of Symbolic Methods in Industrial Analog Circuit Design

Industrial analog circuits are usually designed using numerical simulation tools. To obtain a deeper circuit understanding, symbolic analysis techniques can additionally be applied. Approximation methods which reduce the complexity of symbolic expressions are needed in order to handle industrial-sized problems.
This paper will give an overview to the field of symbolic analog circuit analysis. Starting with a motivation, the state-of-the-art simplification algorithms for linear as well as for nonlinear circuits are presented. The basic ideas behind the different techniques are described, whereas the technical details can be found in the cited references. Finally, the application of linear and nonlinear symbolic analysis will be shown on two example circuits.
Keywords: CAD, automated analog circuit design, symbolic analysis, computer algebra, behavioral modeling, system simulation, circuit sizing, macro modeling, differential-algebraic equations, index
(17 pages, 2003)

45. S. E. Mikhailov, J. Orlik

Asymptotic Homogenisation in Strength and Fatigue Durability Analysis of Composites

Asymptotic homogenisation technique and two-scale convergence is used for analysis of macro-strength and fatigue durability of composites with a periodic structure under cyclic loading. The linear damage accumulation rule is employed in the phenomenological micro-durability conditions (for each component of the composite) under varying cyclic loading. Both local and non-local strength and durability conditions are analysed. The strong convergence of the strength and fatigue damage measure as the structure period tends to zero is proved and their limiting values are estimated.
Keywords: multiscale structures, asymptotic homogenization, strength, fatigue, singularity, non-local conditions
(14 pages, 2003)

46. P. Domínguez-Marín, P. Hansen, N. Mladenović, S. Nickel

Heuristic Procedures for Solving the Discrete Ordered Median Problem

We present two heuristic methods for solving the Discrete Ordered Median Problem (DOMP), for which no such approaches have been developed so far. The DOMP generalizes classical discrete facility location problems, such as the p-median, p-center and Uncapacitated Facility Location problems. The first procedure proposed in this paper is based on a genetic algorithm developed by Moreno Vega [MV96] for p-median and p-center problems. Additionally, a second heuristic approach based on the Variable Neighborhood Search metaheuristic (VNS) proposed by Hansen & Mladenovic [HM97] for the p-median problem is described. An extensive numerical study is presented to show the efficiency of both heuristics and compare them.
Keywords: genetic algorithms, variable neighborhood search, discrete facility location
(31 pages, 2003)

47. N. Boland, P. Domínguez-Marín, S. Nickel, J. Puerto

Exact Procedures for Solving the Discrete Ordered Median Problem

The Discrete Ordered Median Problem (DOMP) generalizes classical discrete location problems, such as the N-median, N-center and Uncapacitated Facility Location problems. It was introduced by Nickel [16], who formulated it as both a nonlinear and a linear integer program. We propose an alternative integer linear programming formulation for the DOMP, discuss relationships between both integer linear programming formulations, and show how properties of optimal solutions can be used to strengthen these formulations. Moreover, we present a specific branch and bound procedure to solve the DOMP more efficiently. We test the integer linear programming formulations and this branch and bound method computationally on randomly generated test problems.
Keywords: discrete location, Integer programming
(41 pages, 2003)

48. S. Feldmann, P. Lang

Padé-like reduction of stable discrete linear systems preserving their stability

A new stability preserving model reduction algorithm for discrete linear SISO-systems based on their impulse response is proposed. Similar to the Padé approximation, an equation system for the Markov parameters involving the Hankel matrix is considered, that here however is chosen to be of very high dimension. Although this equation system therefore in general cannot be solved exactly, it is proved that the approximate solution, computed via the Moore-Penrose inverse, gives rise to a stability preserving reduction scheme, a property that cannot be guaranteed for the Padé approach. Furthermore, the proposed algorithm is compared to another stability preserving reduction approach, namely the balanced truncation method, showing comparable performance of the reduced systems. The balanced truncation method however starts from a state space description of the systems and in general is expected to be more computational demanding.
Keywords: Discrete linear systems, model reduction, stability, Hankel matrix, Stein equation
(16 pages, 2003)

49. J. Kallrath, S. Nickel

A Polynomial Case of the Batch Presorting Problem

This paper presents new theoretical results for a special case of the batch presorting problem (BPSP). We will show tht this case can be solved in polynomial time. Offline and online algorithms are presented for solving the BPSP. Competetive analysis is used for comparing the algorithms.
Keywords: batch presorting problem, online optimization, competetive analysis, polynomial algorithms, logistics
(17 pages, 2003)

50. T. Hanne, H. L. Trinkaus

knowCube for MCDM – Visual and Interactive Support for Multicriteria Decision Making

In this paper, we present a novel multicriteria decision support system (MCDSS), called knowCube, consisting of components for knowledge organization, generation, and navigation. Knowledge organization rests upon a database for managing qualitative and quantitative criteria, together with add-on information. Knowledge generation serves filling the database via e. g. identification, optimization, classification or simulation. For "finding needles in haycocks", the knowledge navigation component supports graphical database retrieval and interactive, goal-oriented problem solving. Navigation "helpers" are, for instance, cascading criteria aggregations, modifiable metrics, ergonomic interfaces, and customizable visualizations. Examples from real-life projects, e.g. in industrial engineering and in the life sciences, illustrate the application of our MCDSS.
Key words: Multicriteria decision making, knowledge management, decision support systems, visual interfaces, interactive navigation, real-life applications.
(26 pages, 2003)

51. O. Iliev, V. Laptev

On Numerical Simulation of Flow Through Oil Filters

This paper concerns numerical simulation of flow through oil filters. Oil filters consist of filter housing (filter box), and a porous filtering medium, which completely separates the inlet from the outlet. We discuss mathematical models, describing coupled flows in the pure liquid subregions and in the porous filter media, as well as interface conditions between them. Further, we reformulate the problem in fictitious regions method manner, and discuss peculiarities of the numerical algorithm in solving the coupled system. Next, we show numerical results, validating the model and the

algorithm. Finally, we present results from simulation of 3-D oil flow through a real car filter.
Keywords: oil filters, coupled flow in plain and porous media, Navier-Stokes, Brinkman, numerical simulation
(8 pages, 2003)

52. W. Dörfler, O. Iliev, D. Stoyanov, D. Vassileva
On a Multigrid Adaptive Refinement Solver for Saturated Non-Newtonian Flow in Porous Media

A multigrid adaptive refinement algorithm for non-Newtonian flow in porous media is presented. The saturated flow of a non-Newtonian fluid is described by the continuity equation and the generalized Darcy law. The resulting second order nonlinear elliptic equation is discretized by a finite volume method on a cell-centered grid. A nonlinear full-multigrid, full-approximation-storage algorithm is implemented. As a smoother, a single grid solver based on Picard linearization and Gauss-Seidel relaxation is used. Further, a local refinement multigrid algorithm on a composite grid is developed. A residual based error indicator is used in the adaptive refinement criterion. A special implementation approach is used, which allows us to perform unstructured local refinement in conjunction with the finite volume discretization. Several results from numerical experiments are presented in order to examine the performance of the solver.
Keywords: Nonlinear multigrid, adaptive refinement, non-Newtonian flow in porous media
(17 pages, 2003)

53. S. Kruse
On the Pricing of Forward Starting Options under Stochastic Volatility

We consider the problem of pricing European forward starting options in the presence of stochastic volatility. By performing a change of measure using the asset price at the time of strike determination as a numeraire, we derive a closed-form solution based on Heston's model of stochastic volatility.
Keywords: Option pricing, forward starting options, Heston model, stochastic volatility, cliquet options
(11 pages, 2003)

54. O. Iliev, D. Stoyanov
Multigrid – adaptive local refinement solver for incompressible flows

A non-linear multigrid solver for incompressible Navier-Stokes equations, exploiting finite volume discretization of the equations, is extended by adaptive local refinement. The multigrid is the outer iterative cycle, while the SIMPLE algorithm is used as a smoothing procedure. Error indicators are used to define the refinement subdomain. A special implementation approach is used, which allows to perform unstructured local refinement in conjunction with the finite volume discretization. The multigrid - adaptive local refinement algorithm is tested on 2D Poisson equation and further is applied to a lid-driven flows in a cavity (2D and 3D case), comparing the results with bench-mark data. The software design principles of the solver are also discussed.
Keywords: Navier-Stokes equations, incompressible flow, projection-type splitting, SIMPLE, multigrid methods, adaptive local refinement, lid-driven flow in a cavity
(37 pages, 2003)

55. V. Starikovicius
The multiphase flow and heat transfer in porous media

In first part of this work, summaries of traditional Multiphase Flow Model and more recent Multiphase Mixture Model are presented. Attention is being paid to attempts include various heterogeneous aspects into models. In second part, MMM based differential model for two-phase immiscible flow in porous media is considered. A numerical scheme based on the sequential solution procedure and control volume based finite difference schemes for the pressure and saturation-conservation equations is developed. A computer simulator is built, which exploits object-oriented programming techniques. Numerical result for several test problems are reported.
Keywords: Two-phase flow in porous media, various formulations, global pressure, multiphase mixture model, numerical simulation
(30 pages, 2003)

56. P. Lang, A. Sarishvili, A. Wirsen
Blocked neural networks for knowledge extraction in the software development process

One of the main goals of an organization developing software is to increase the quality of the software while at the same time to decrease the costs and the duration of the development process. To achieve this, various decisions e.ecting this goal before and during the development process have to be made by the managers. One appropriate tool for decision support are simulation models of the software life cycle, which also help to understand the dynamics of the software development process. Building up a simulation model requires a mathematical description of the interactions between di.erent objects involved in the development process. Based on experimental data, techniques from the .eld of knowledge discovery can be used to quantify these interactions and to generate new process knowledge based on the analysis of the determined relationships. In this paper blocked neuronal networks and related relevance measures will be presented as an appropriate tool for quanti.cation and validation of qualitatively known dependencies in the software development process.
Keywords: Blocked Neural Networks, Nonlinear Regression, Knowledge Extraction, Code Inspection
(21 pages, 2003)

57. H. Knaf, P. Lang, S. Zeiser
Diagnosis aiding in Regulation Thermography using Fuzzy Logic

The objective of the present article is to give an overview of an application of Fuzzy Logic in Regulation Thermography, a method of medical diagnosis support. An introduction to this method of the complementary medical science based on temperature measurements – so-called thermograms – is provided. The process of modelling the physician's thermogram evaluation rules using the calculus of Fuzzy Logic is explained.
Keywords: fuzzy logic,knowledge representation, expert system
(22 pages, 2003)

58. M.T. Melo, S. Nickel, F. Saldanha da Gama
Largescale models for dynamic multi-commodity capacitated facility location

In this paper we focus on the strategic design of supply chain networks. We propose a mathematical modeling framework that captures many practical aspects of network design problems simultaneously but which have not received adequate attention in the literature. The aspects considered include: dynamic planning horizon, generic supply chain network structure, external supply of materials, inventory opportunities for goods, distribution of commodities, facility configuration, availability of capital for investments, and storage limitations. Moreover, network configuration decisions concerning the gradual relocation of facilities over the planning horizon are considered. To cope with fluctuating demands, capacity expansion and reduction scenarios are also analyzed as well as modular capacity shifts. The relation of the proposed modeling framework with existing models is discussed. For problems of reasonable size we report on our computational experience with standard mathematical programming software. In particular, useful insights on the impact of various factors on network design decisions are provided.
Keywords: supply chain management, strategic planning, dynamic location, modeling
(40 pages, 2003)

Status quo: November 2003