Derivative Contracts as Active Documents

ComDeCo Project
Dependable Adaptive Systems and Mathematical Modelling (DASMOD)
Cluster of Excellence in Rhineland Palatinate, Germany

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Objectives

The interdisciplinary joint work of

- Software Technology Group, University of Kaiserslautern
- Financial Mathematics and Stochastic Controlling Group, University of Kaiserslautern
- Department of Financial Mathematics, Fraunhofer ITWM, Kaiserslautern

Aims at providing a conceptual and technological framework for derivative contract design and valuation

Fraunhofer ITWM: An Overview

- **Fraunhofer**: Biggest german society for applied research
- **ITWM:** Industrial and financial mathematics

  7+2 Departments (appr. 140 Employees)
- **FhG Mix:** 30-40% FhG Public funding, rest: projects
  - 50 % Public Projects (EU, DFG, Rheinlandpfalz-Stiftung für Innovationen)
  - 50 % Projects with partners from industry
- **Department of financial mathematics:** 80% projects

  with partners from financial industry

The three pillars of Financial Engineering

Financial Engineering

Banking/ Finance

Economics
Quantitative Methods
Trading/Sales
Financial products

IT

C,C++, VBA
Java, html, xml
Databases (SQL)

Financial mathematics

Stochastic Processes
ODEs/PDEs
Numerics
Statistics

Part 1

Motivation

Why Derivative Contracts should be Active Documents

Plain vanilla options

European options

The right (but not the obligation!) to buy (Call) or sell (Put)

an amount of a definite

Underlying S

with some spot price S(t) at some expiry date

Maturity T

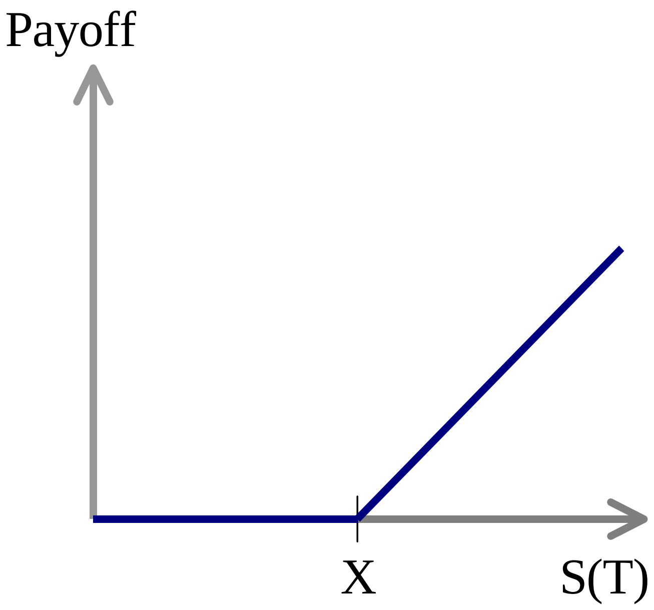
for a prescribed price

Strike X

Pay Off Function

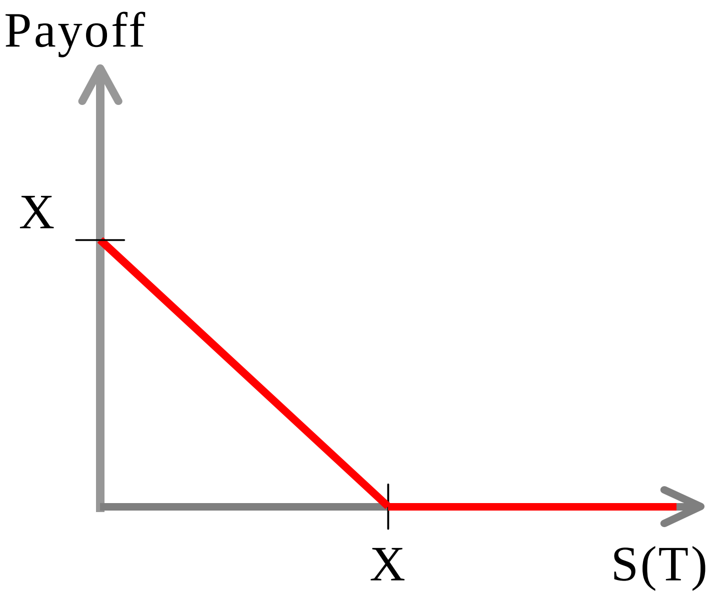
Plain Vanilla European Call

$$C(S(T), X) = \max[0, S(T) - X]$$



Plain Vanilla European Put

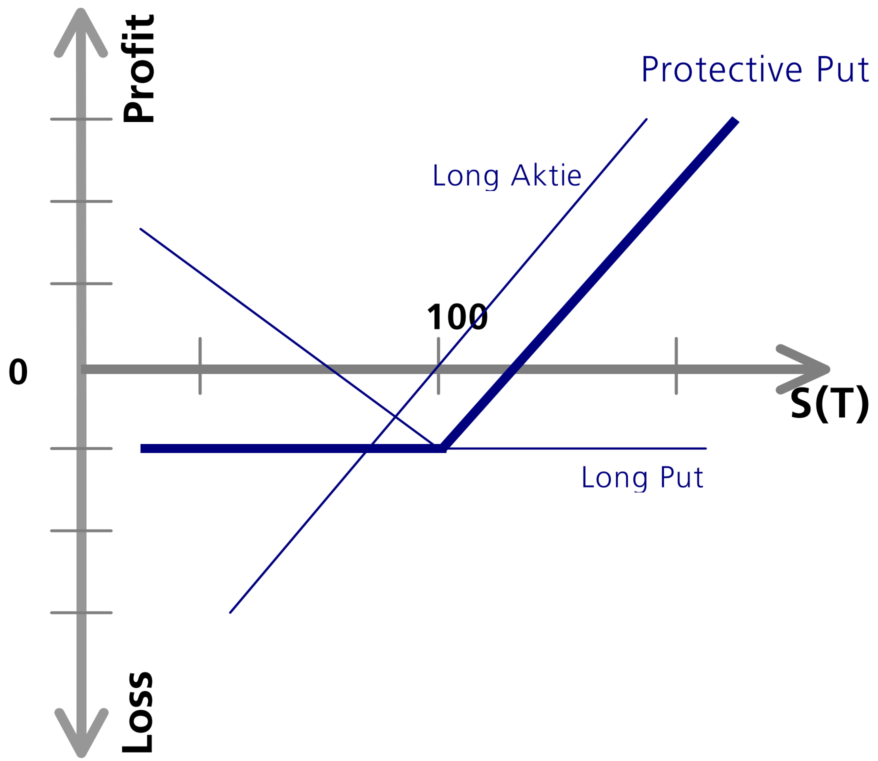
$$P(S(T), X) = \max[0, X - S(T)]$$



Advanced option trading in the past

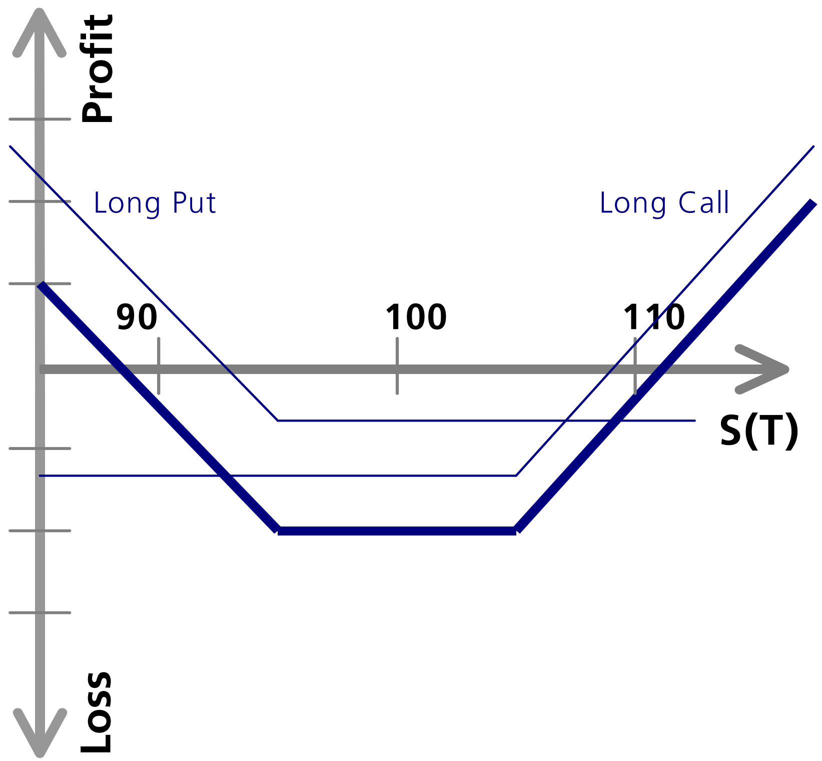
Protective Put
Long Underlying und Long Put

Payoff



Long Strangle
Long 105 Call für 15 EUR
Long 95 Put für 10 EUR

Payoff



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Exotics and structured products today

Capped-Floored-Asian Option with Lookback Feature

$$\text{Payoff} = \text{NA} \times \text{Max}\left[0; \frac{1}{m}\left(\sum_{i=1}^{n} \min\left\{Cap, \max\left(\frac{S(t_i)}{S(T_0)}, Floor\right)\right\} \times f(i)\right) - K\right]$$

$$f(i) = \begin{cases} 1 \text{ if } i > 1 \text{ and } S(t_i) \geq a \times S(t_{i-1}) \text{ and } S(t_i) \geq b \times \max S(t_k) \text{ and } S(t_i) \geq c \times S(T_0) \\ 1 \text{ if } i = 1 \text{ and } S(t_i) \geq d \times S(T_0) \\ 0 \text{ otherwise} \end{cases}$$

$$m = \sum_{i=1}^{n} f(i) \quad .$$

High risk: Some big losses on derivatives

- **February 1995:** Barings Bank announces a loss which ultimately totals **$1.38 billion**, related to derivatives trading in Singapore by Nickolas Leeson

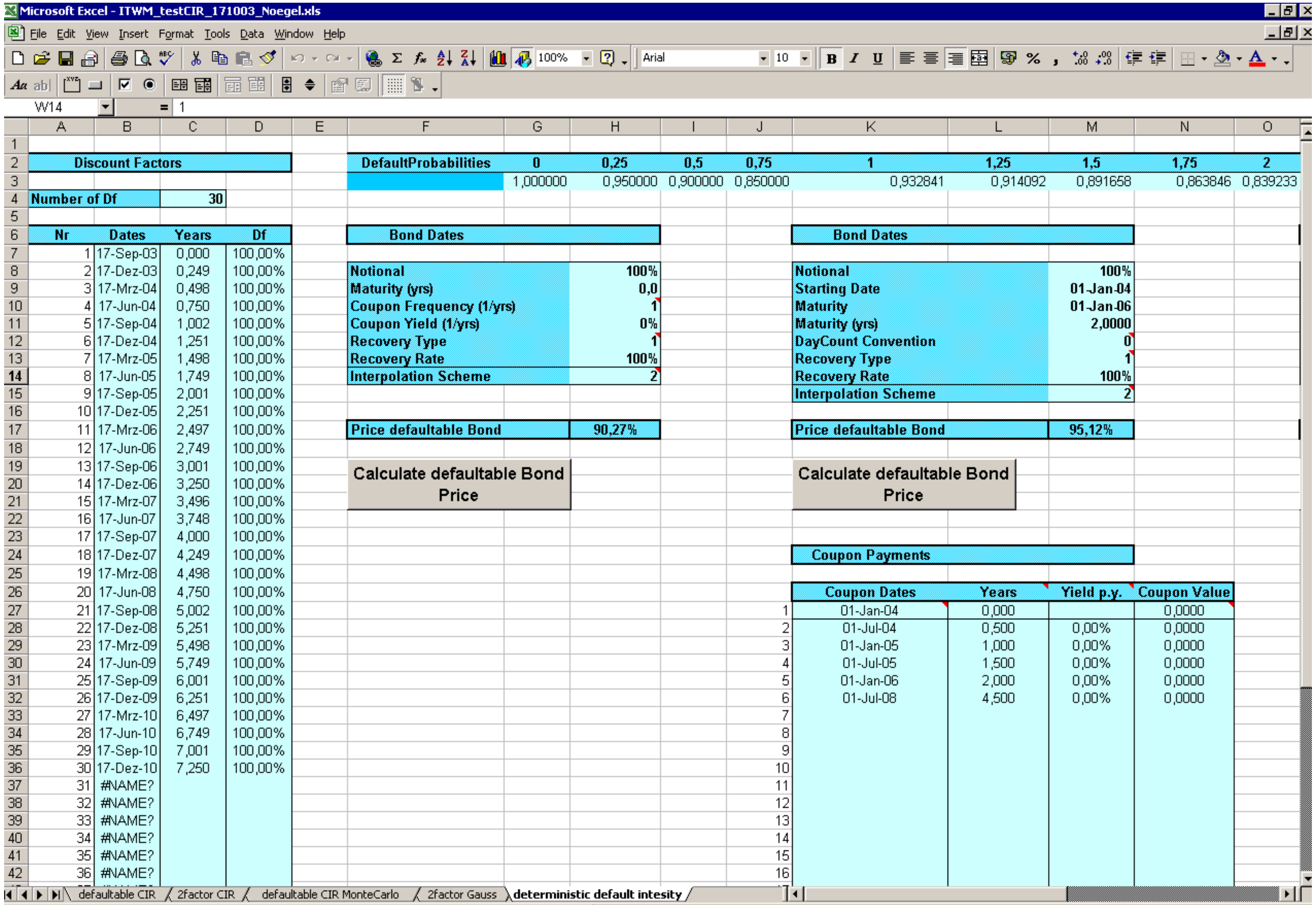
- **December 1996:** NatWest Bank finds losses of **77 million GPS** by mispricing derivatives

- **November 1997:** Chase Manhatan have lost up to **$200 million** on trading emerging-market depts through complex derivatives products

- **January 2002:** Allied Irish Banks incurs a **$750 million** loss from FX trades by rogue trader John Rusnak

Source: Dan Atkinson. USB Pledged Derivatives Explanation. *Manchester Guardian.*

State of Practice: Excel Prototypes as final products



- VBA fragments as glue code
- Native language DLLs

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Problems

- Inflexibility
- Complex Maintainance
- Archaic Design
- Proprietary Formats
- Financial Engineers are usually **not** Software Engineers

But

- Both have:

  Pressure to deliver high quality products in fast changing markets

But: Pay-offs as Components

- Forward-starting options

$$C_{FS} = \max\left(\frac{S(t_2)}{S(t_1)} - X, 0\right) \qquad 0 \le t_1 \le t_2$$

- Globally-floored/capped cliquets

$$C_{GFCC} = \max\left(\sum_{i=1}^{n} \max\left[\min\left\{\frac{S(t_i) - S(t_{i-1})}{S(t_{i-1})}, C\right\}, F\right], 0\right) \quad 0 \le t_1 \cdots \le t_n$$

XML based solution using software technology

The past:

- Spreadsheet interface,
- Business logic (C++, VB, Java...)
- SQL database

The future:

- XML based solution
- Component-Orientation
- Higher Order Logic

Advantages:

- Open standard
- Encapsulation and scalability
- Cross platform
- Hidden complexity
- Quicker development cycle
- Available as web service

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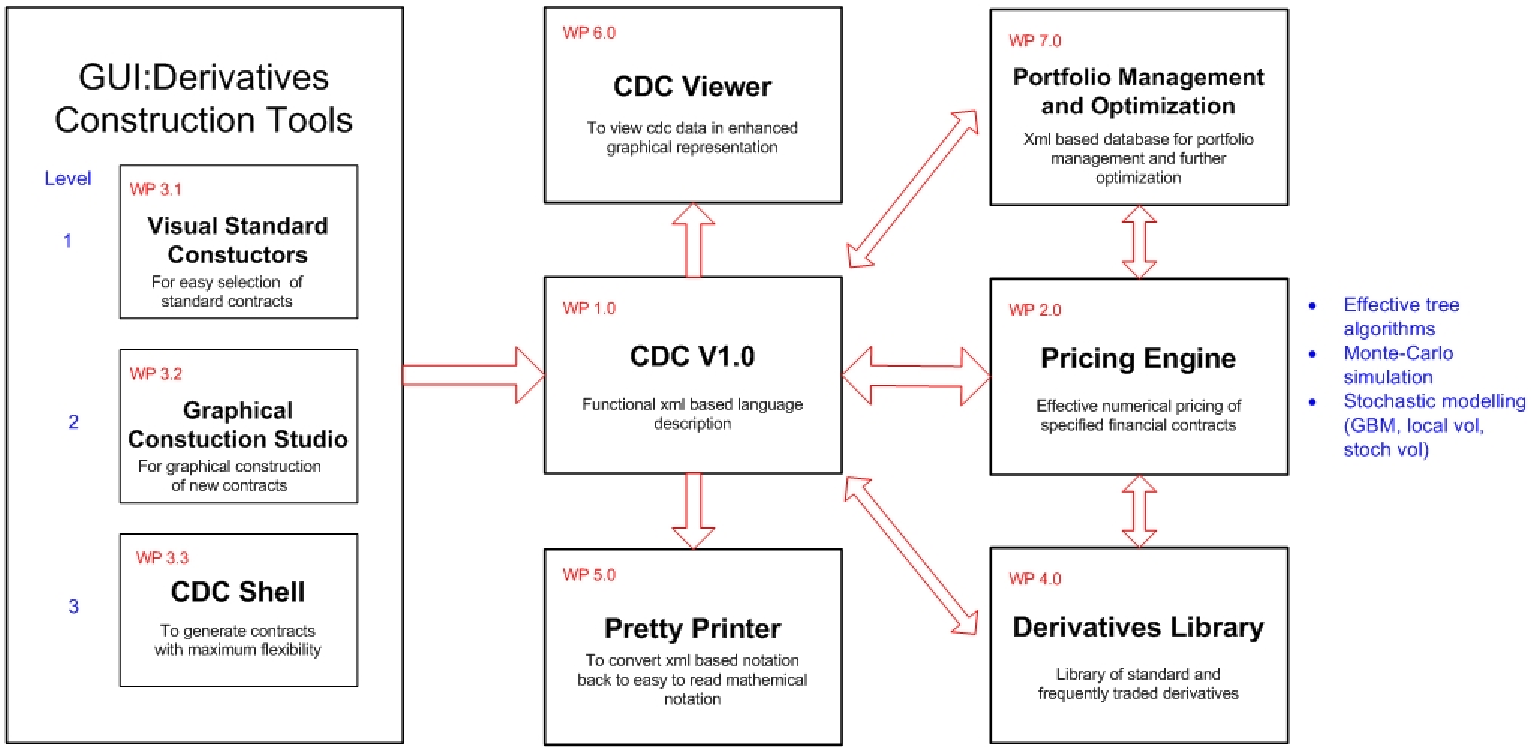
ComDeCo: Best practice and standard technology

Project Structure: ComDeCo

Friday, March 10, 2006



CDC = ComDeCo = Composable Derivative Contracts

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Components and Active Documents

Components: well tested building blocks (develop one, use many times)

Problem: Audience of current technologies as COM, CCM or JavaBeans are developers, not end users!

Active Documents = dynamic (!) combination of components

- End-user compatible conceptual framework
- Enhancement of the well-known document metaphor
  - Computer based Hyperdocument
  - Explorative and guided (Drag&Drop) composition (Decorator Pattern)

Composition Constraints

- **Constraints** checked just in time and on demand
  - guiding the user
  - playing with the contracts. "What if…?"
- **Structural Constraints** (overall "architecture")
  - XML Schema, Relax NG, Schematron
  - Graph-based specification techniques
- **Semantical Constraints** (beyond pure structural)
  - First Order Logic
  - Forward Chaining Rule Engines

Component Model: What does it look like?

- XML representation → Active Documents

```
<derivative>
   <sell>
      <condition>
          <at timestep = 7/>
      </condition>
      <observable id=foo model=FOO/>
   </sell>
</derivative>
```

- Components with local and global constraint fragments
- Fragments aggregated by Runtime System
- Contracts are queryable entities
- Java → Platform independent

Part 2

The Models

Black Scholes, Local Volatility, Stochastic Volatility

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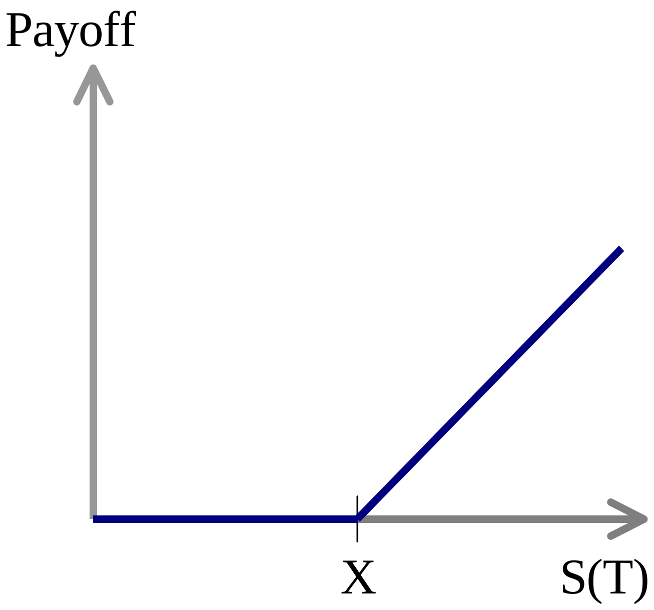
Black-Scholes as (former) market standard

Plain vanilla european Call

$$C(S(T),K) = \max[0, S(T) - K]$$



Black-Scholes formular

$$C = S(t)N(d_1) - e^{-r\tau} KN(d_2)$$

$$d_1 = \frac{1}{\sigma\sqrt{\tau}}[\log(S/X) + r] + \frac{1}{2}\sigma\sqrt{\tau}$$

$$d_2 = d_1 - \sigma\sqrt{\tau}$$

$\tau \equiv T - t$	Time to maturity
r	Risk free interest rate
σ	Volatility

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Black-Scholes theory: assumptions

- No credit risk, only market risk
- The market is maximally efficient
- No transaction costs
- The market is arbitrage-free
- The underlying is arbitrarily divisible
- Continuous trading/hedging is possible
- The time evolution of the underlying is stochastic and exhibits a geometric Brownian motion.
- Interest rate, dividend yield and volatility are constant

Black-Scholes theory

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t)$$

Black-Scholes SDE

$\mu = r - q$ drift r: risik-free interest rate

q: dividend yield

$W(t)$ Brownian motion

σ constant volatility

Black-Scholes theory 2

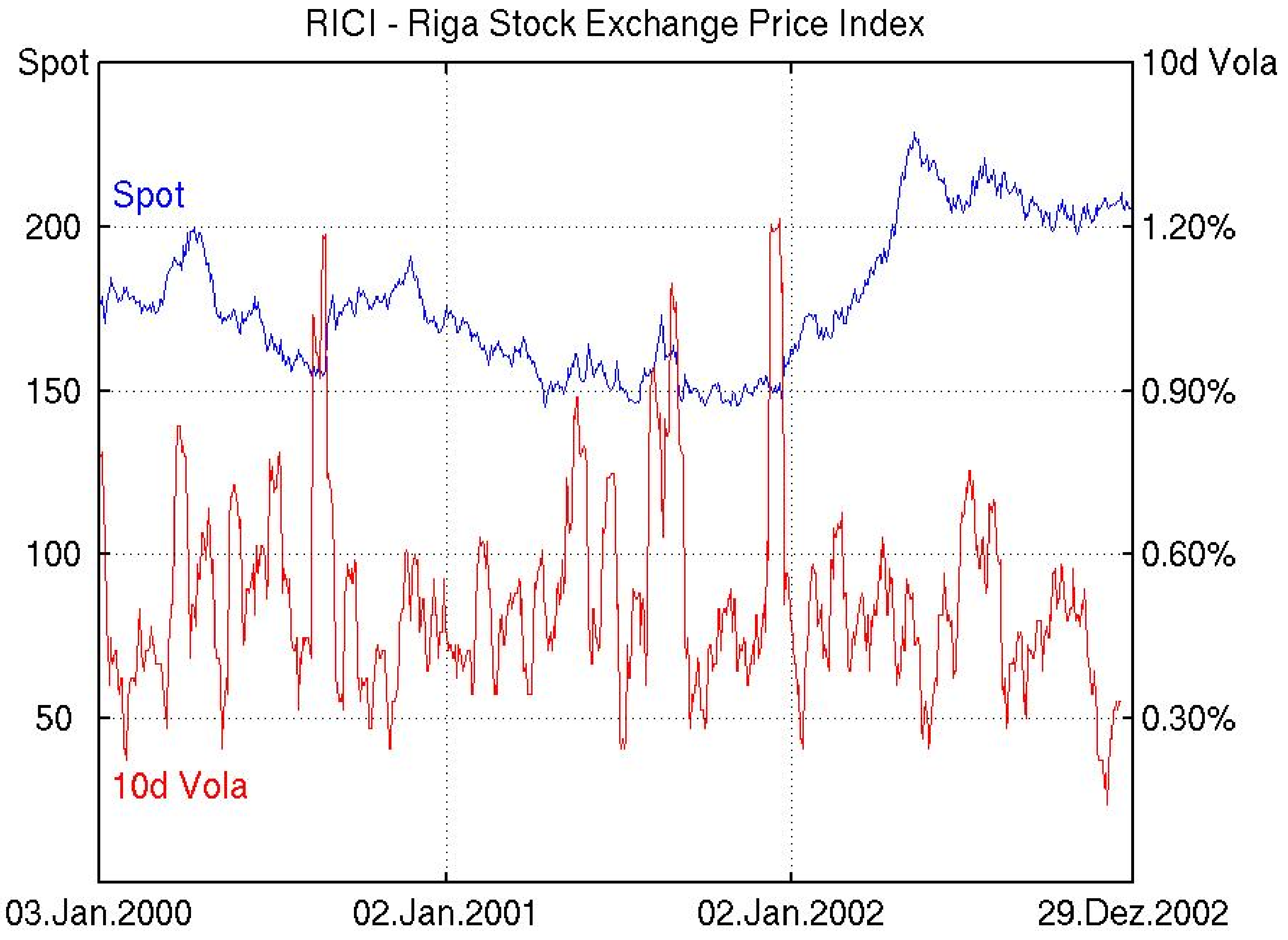
PRO

- Simple model
- Closed-form solution
- Simple numerical implementation
- Closed-form solutions for "Greeks" $(\Delta = \partial C / \partial S, \ldots)$
- (former) Market standard !

CONTRA

- Defects in the assumptions !
  - Volatility **not** constant!
  - Current volatility difficult to determine
  - Struktured Products very sensitive to forward vol!
  - The world is not log-normal

Historic volatility



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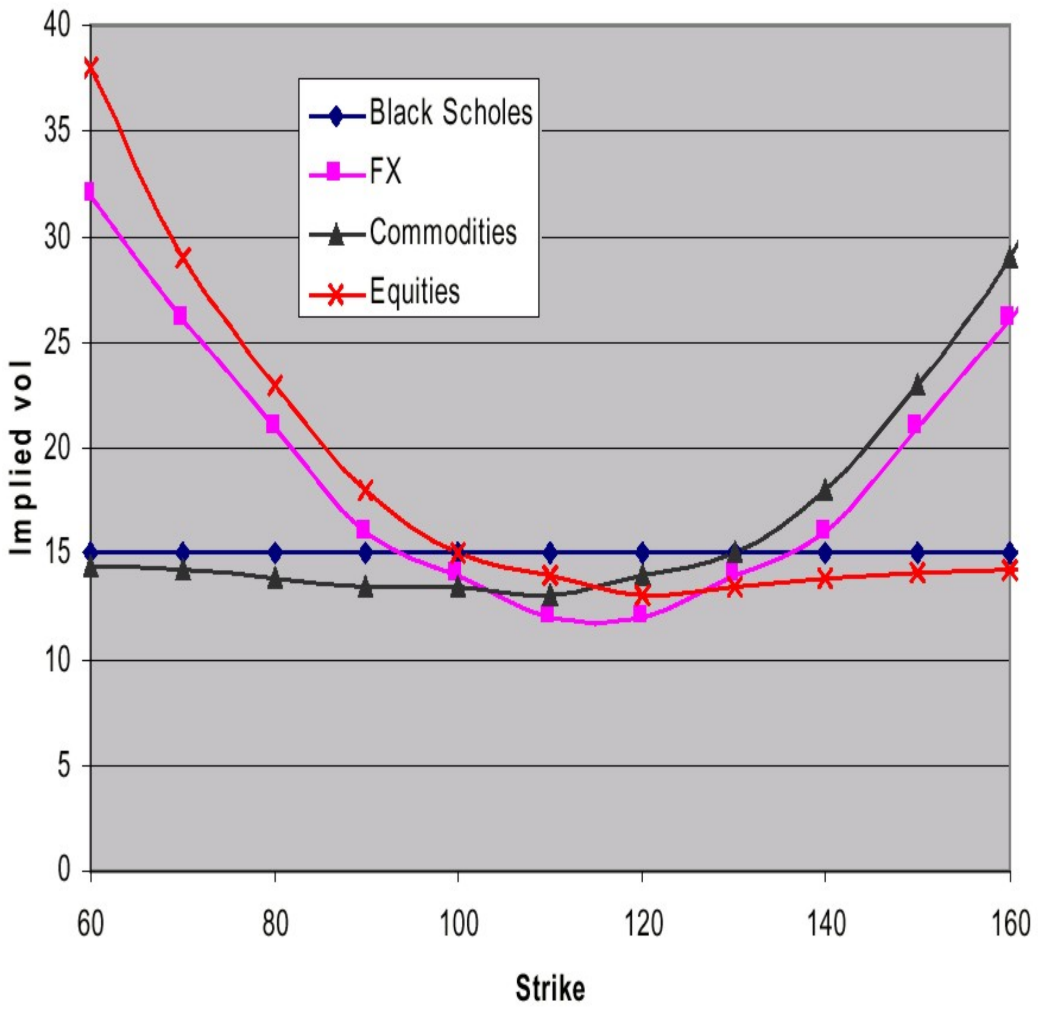
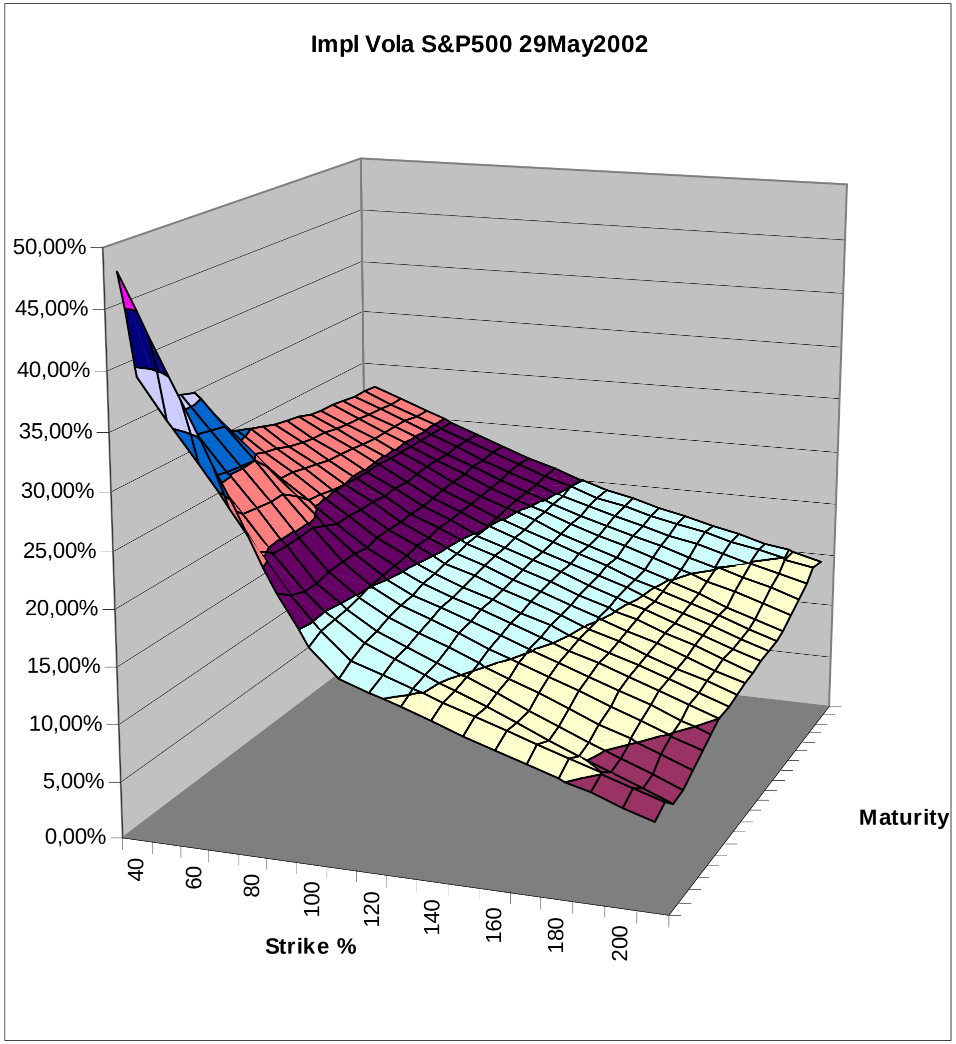
Black Scholes doesn't fit the volatility smile





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Local Volatility Models

$$dS(t) = \mu S(t)dt + \sigma(S,t)S(t)dW(t)$$

local volatility: $\sigma(S,t)$

Famous result (Dupire)

$$\sigma_{imp}^2(S,t) = 2\frac{-\frac{\partial C}{\partial t} - \mu\left(C - S\frac{\partial C}{\partial K}\Big|_{K=S}\right)}{S^2\frac{\partial^2 C}{\partial K^2}\Big|_{K=S}}$$

Limited number of strikes and maturities ➡ **ill posed problem !**

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Stochastic volatility models

$$dS(t) = \mu(t)S(t)dt + \sigma(t)S(t)dW(t)$$

continuous-time models

- GARCH diffusion model
- 3/2 model
- Square root model
  e.g. Heston (1993)

instantaneous volatility not messurable!

$\Longrightarrow$ calibration to market data!

discrete-time models

- GARCH models
  - Duan (1995)
  - Heston und Nandi (1997)

volatility from historic time series!

$\Longrightarrow$ calibration with MLL estimation

Heston model

$$dS(t) = (\mu + \lambda)S(t)dt + \sqrt{v(t)}S(t)dW_1(t)$$

$$dv(t) = \kappa(\theta - v(t))dt - \lambda v(t)dt + \sqrt{v(t)}\sigma dW_2(t)$$

$dW_i(t)$ Wiener processes with correlation $dW_1 dW_2 = \rho dt$

μ drift

λ market price of risk

κ mean reversion speed

θ mean reversion level

σ volatility of volatility

risk-neutral measure

$\Rightarrow$

$\mu = r - q$

$\kappa^* = \kappa + \lambda \qquad \theta^* = \dfrac{\kappa\theta}{\kappa + \lambda}$

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Competence on Stochastic Volatility

- U.Nögel and S.Mikhailov. Heston's stochastic volatility model. Implementation, calibration and some extensions. To be published in: P. Wilmott, *The Best of Wilmott Vol.I,* Wiley

- S.Kruse and U.Nögel. On Pricing of Forward Starting Options in Heston's Model on Stochastic Volatility. *Finance and Stochastics, 9,233-250(2005)*

- U.Nögel and S.Mikhailov. Heston's stochastic volatility model. Implementation, calibration and some extensions. *WILMOTT Magazine, July* 2003, 74-79

- U.Nögel. Pricing cliquet options using stochastic volatility models. Selected Proceedings of International Scientific Conference *Information Society and Modern Business. Jan. 31-Feb. 1, 2003 Ventspils* 2004

- U.Nögel. Option pricing using stochastic volatility. *Progress in Industrial Mathematics at ECMI 2002.* Springer Verlag, Heidelberg 2004

Pricing of exotic options

liquid market

$C_1, C_2, \ldots, C_N$

traded options

calibration

model

$p = (p_1, p_2, \ldots, p_N)$

parameters

$C_1(p), C_2(p), \ldots C_N(p)$

model prices

exotic option

$C_{GCFC} = ?$

MCSimulation

Tree Algorithm

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What we've done so far

Software-Technology

- Conceptual foundations of derivative construction tool chain
- First version of functional XML based language description (CDC V1.0)
- CDC specification of common plain vanilla stock options
- Developent of prototype for visual composition of derivates contracts

Financial Mathematics

- Implementation of Binomial tree in the existing set-up
- First prototype of efficient multi-dimensional tree algorithms
- Conceptual proposal of use of further models beyond Black-Scholes

In the Future…

- Component Model improvements and refinements
- Composition Facility enhancements using bytecode inspection
- Investigating possibilities of an OpenOffice Legacy Bridge
- Investigation and implementation of further effective tree algorithms
- Integration of further models and numerical algorithms (MC)
- Extending the ComDeCo framework to Credit Derivatives (BDS, CDO)

Futher Informations

- Project Homepage

  http://www.dasmod.de/~comdeco

- Contact

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Academic **and** industrial cooperations are welcome!

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